

Towards the chiral critical surface of QCD

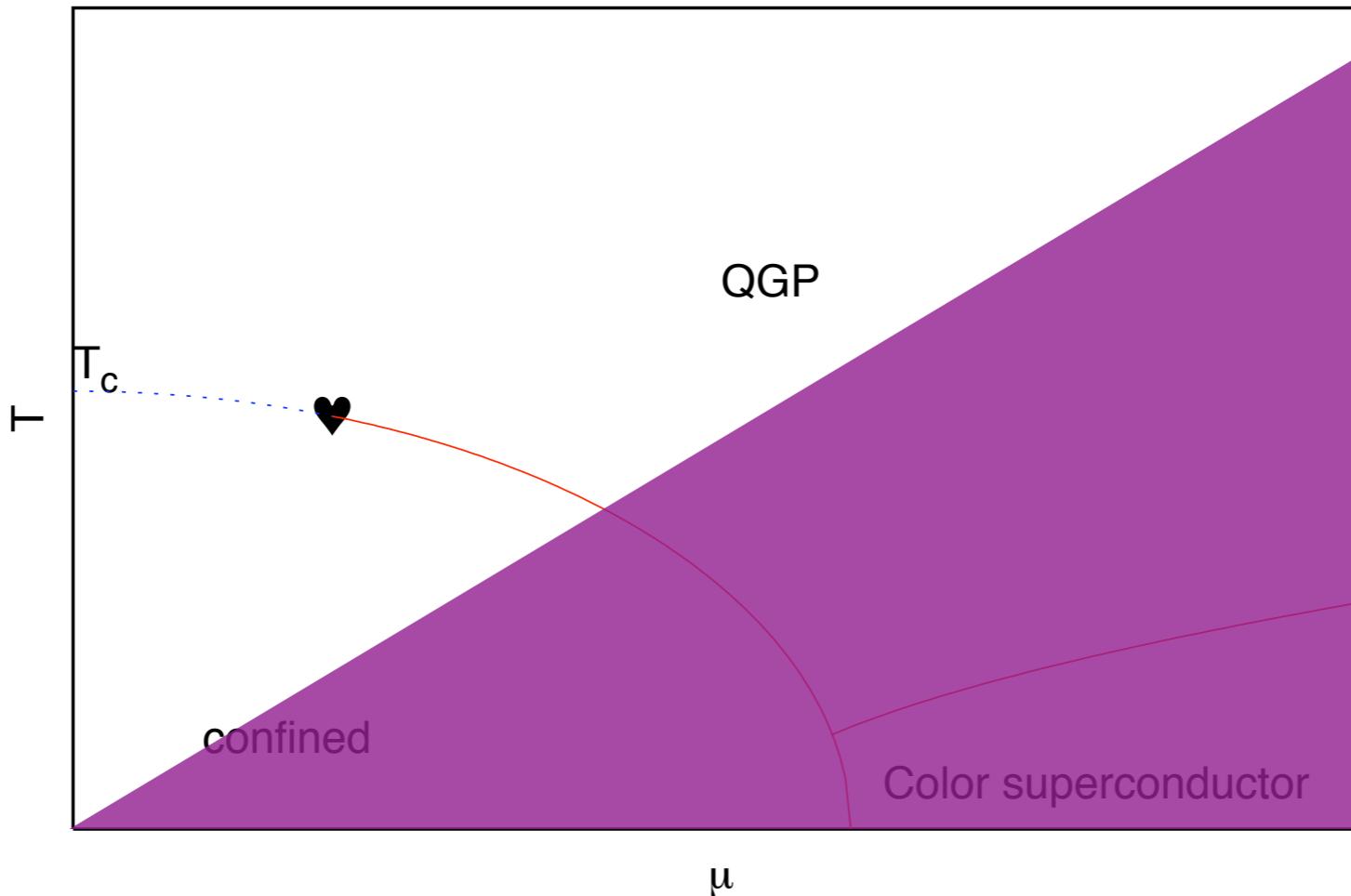
Owe Philipsen



- Is there a critical end point in the QCD phase diagram?
- Is it connected to a chiral phase transition?

in collaboration with Ph. de Forcrand (ETH/CERN): PoS LAT08:208; JHEP 0811:012

The calculable region of the phase diagram



- 2001-present: sign problem not solved, circumvented by approximate methods: **reweighting, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1$** ($\mu = \mu_B/3$)
- Upper region: equation of state, screening masses, quark number susceptibilities etc. under control
- Here: phase diagram itself, **most difficult!**

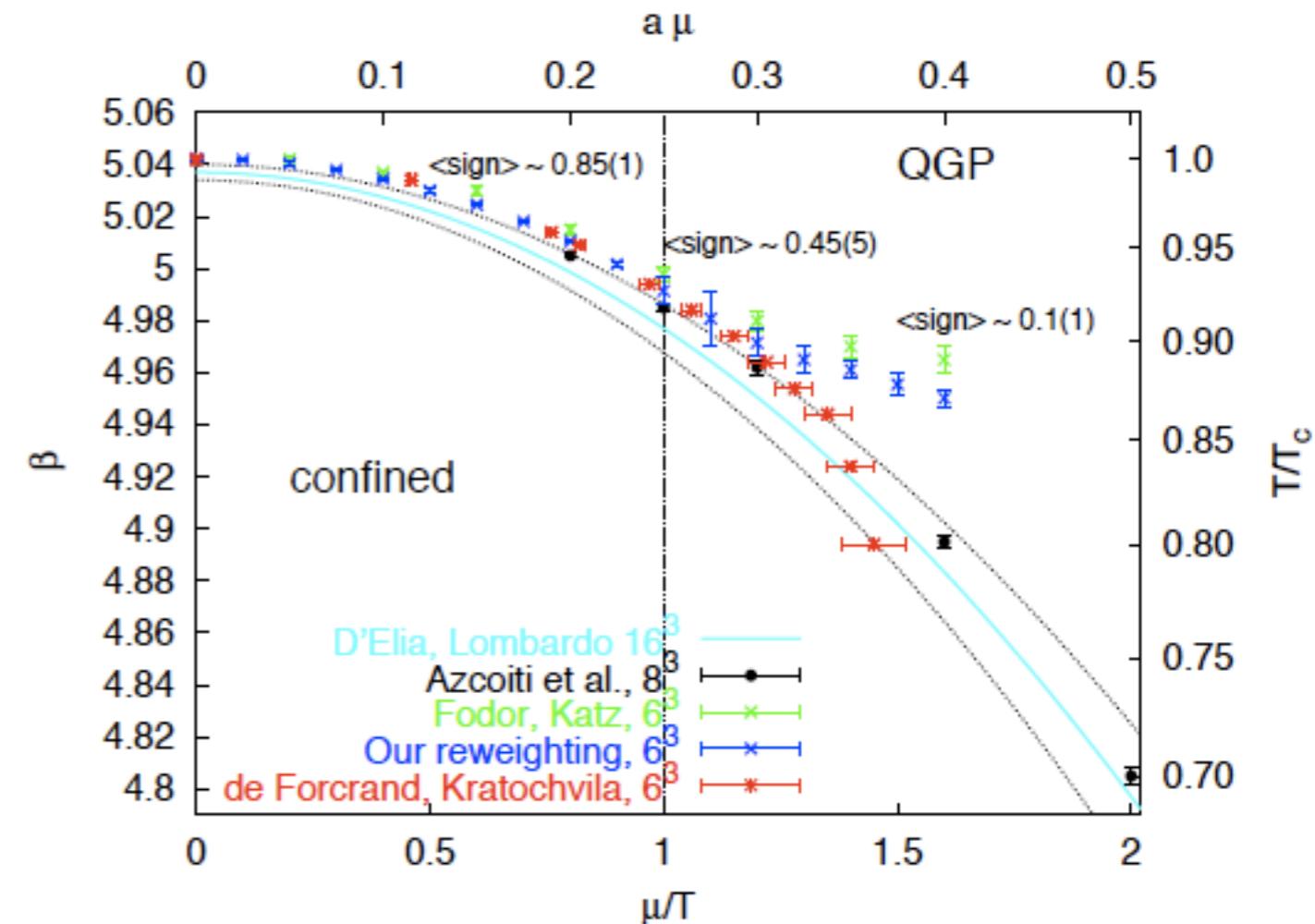
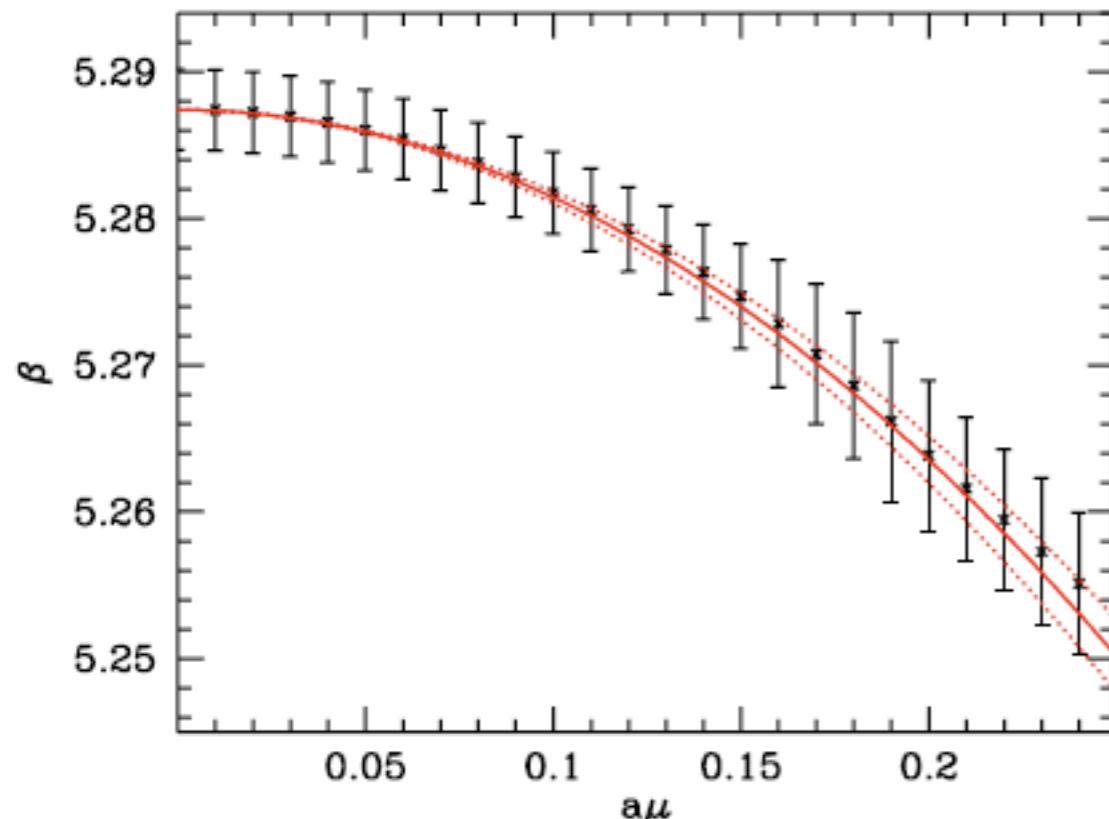
Comparing different approaches: $N_f = 2$ (left), $N_f = 4$ (right):

...with same action (unimproved staggered), same mass, $Nt=4$

Reweighting vs. imag. μ (FK, FP)

Rew., imag. μ , canonical ensemble ...

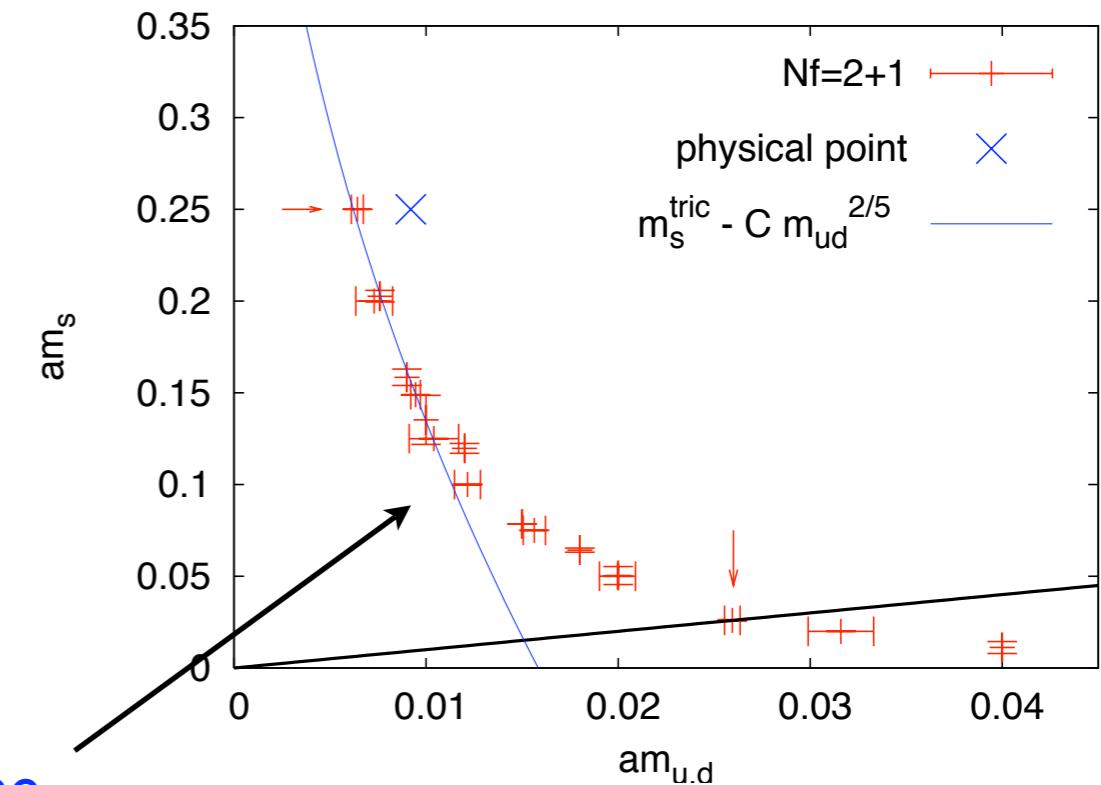
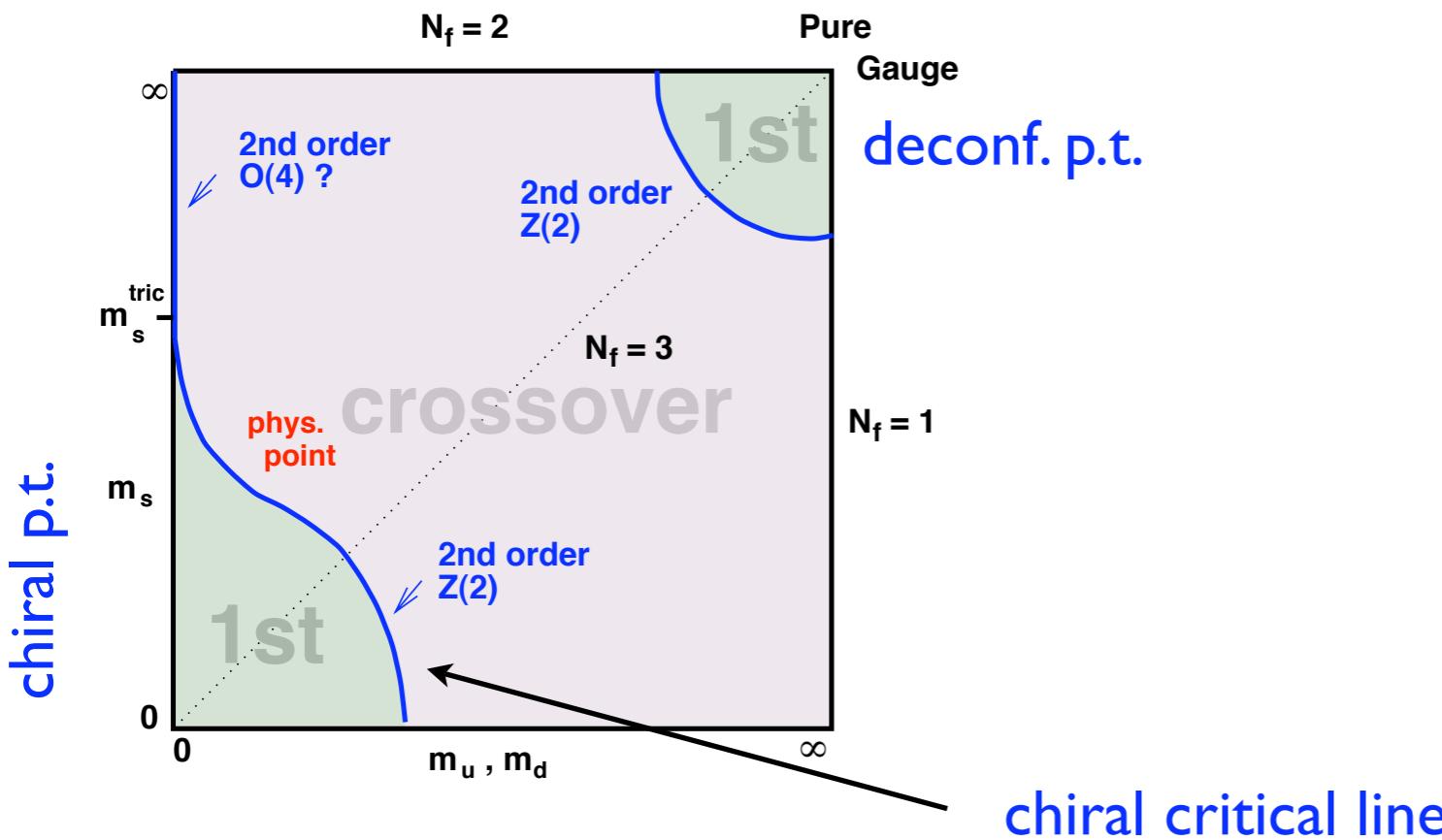
de Forcrand, Kratochvila 05



All agree on $T_0(m, \mu)!!!$

$(\mu/T \lesssim 1)$

Hard part: order of p.t., arbitrary quark masses $\mu = 0$



- physical point: crossover in the continuum

Aoki et al 06

- chiral critical line on $N_t = 4, a \sim 0.3$ fm

de Forcrand, O.P. 07

- consistent with tri-critical point at $m_{u,d} = 0, m_s^{\text{tric}} \sim 2.8T$

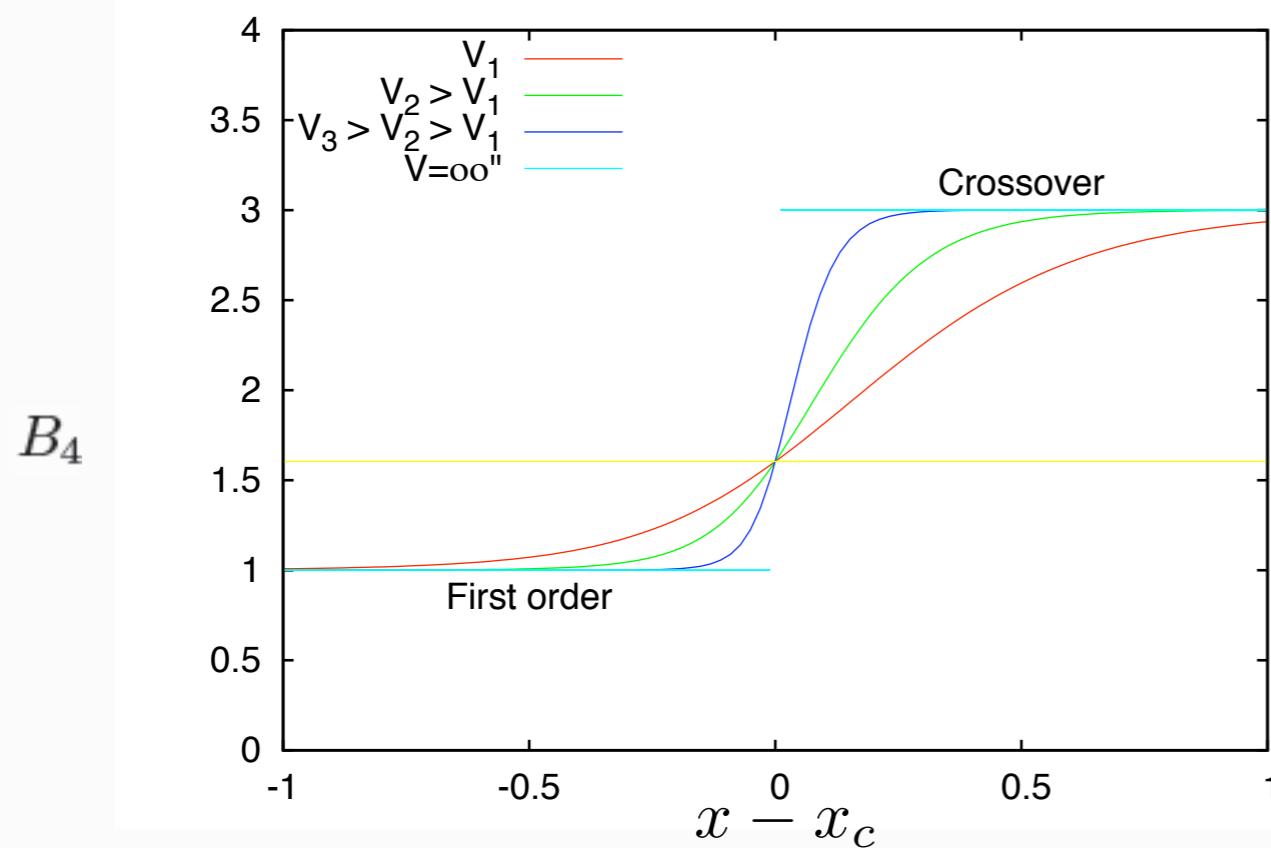
- But: $N_f = 2$ chiral O(4) vs. 1st still open
 $U_A(1)$ anomaly!

Di Giacomo et al 05, Kogut, Sinclair 07
Chandrasekharan, Mehta 07

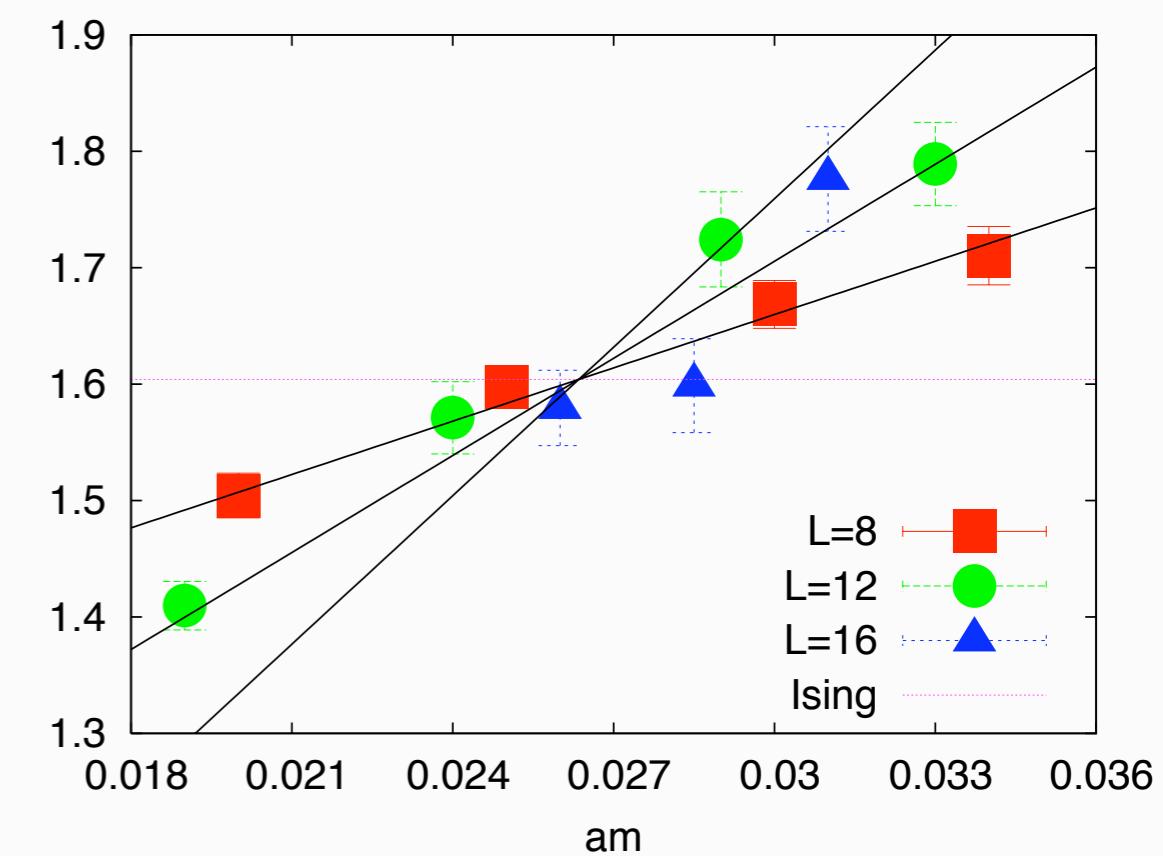
How to identify the order of the phase transition

$$B_4(\bar{\psi}\psi) \equiv \frac{\langle (\delta\bar{\psi}\psi)^4 \rangle}{\langle (\delta\bar{\psi}\psi)^2 \rangle^2} \xrightarrow{V \rightarrow \infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first - order} \\ 3 & \text{crossover} \end{cases}$$

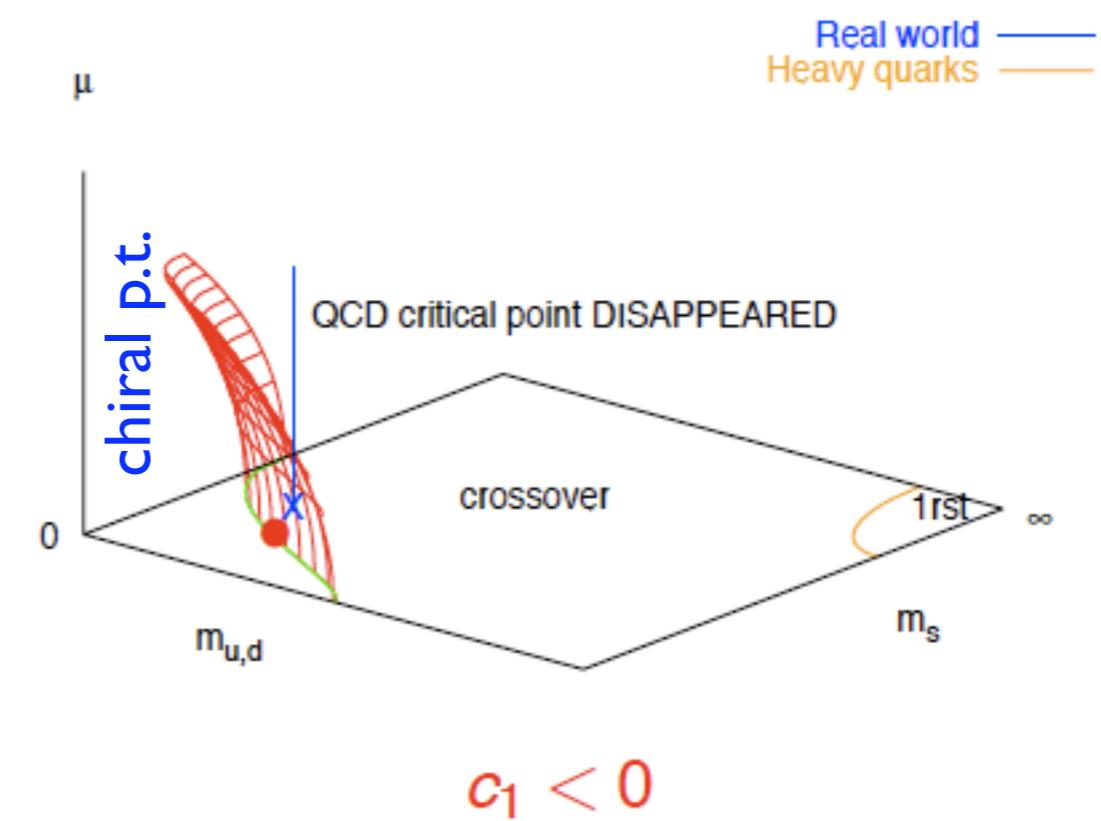
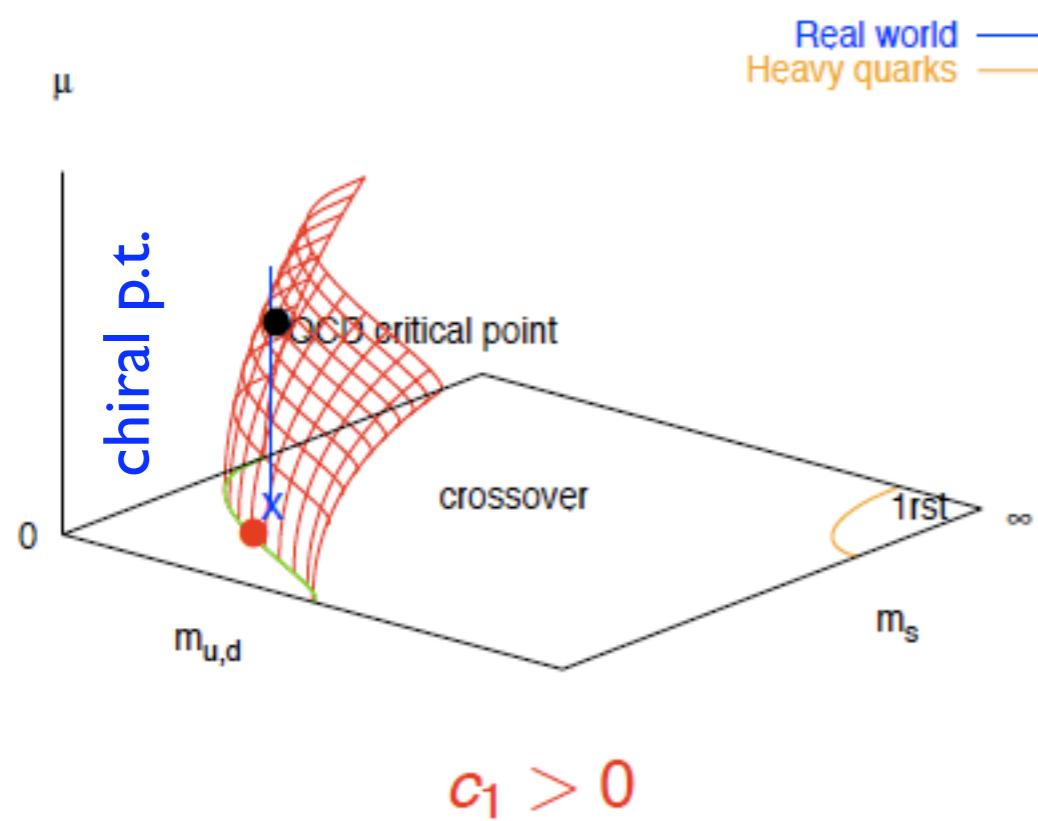
$\mu = 0 :$ $B_4(m, L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$



parameter along phase boundary, $T = T_c(x)$



$\mu \neq 0$, conservative: follow chiral critical line → surface



$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T} \right)^{2k}$$

$$\frac{d am^c}{d(a\mu)^2} = - \frac{\partial B_4}{\partial(a\mu)^2} / \frac{\partial B_4}{\partial am}$$

hard/easy

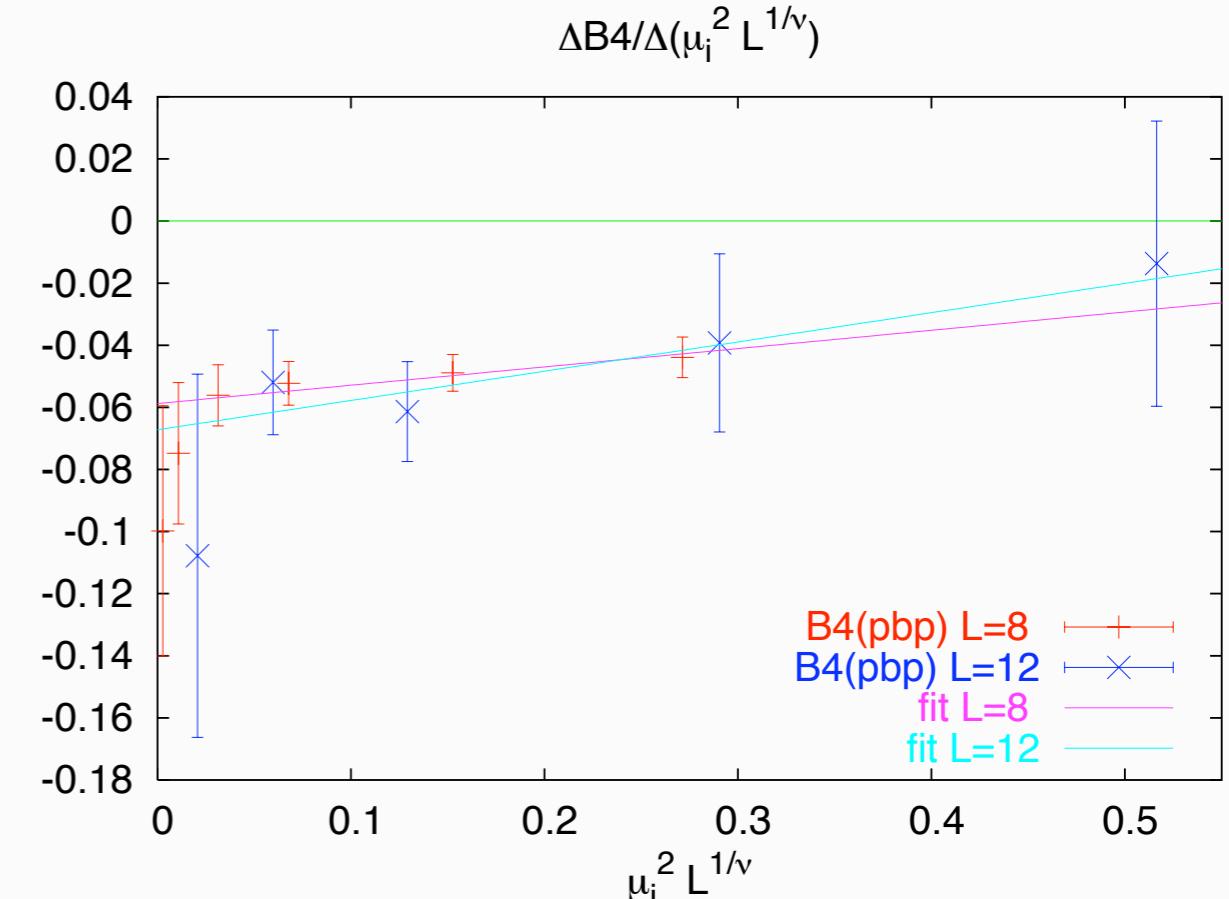
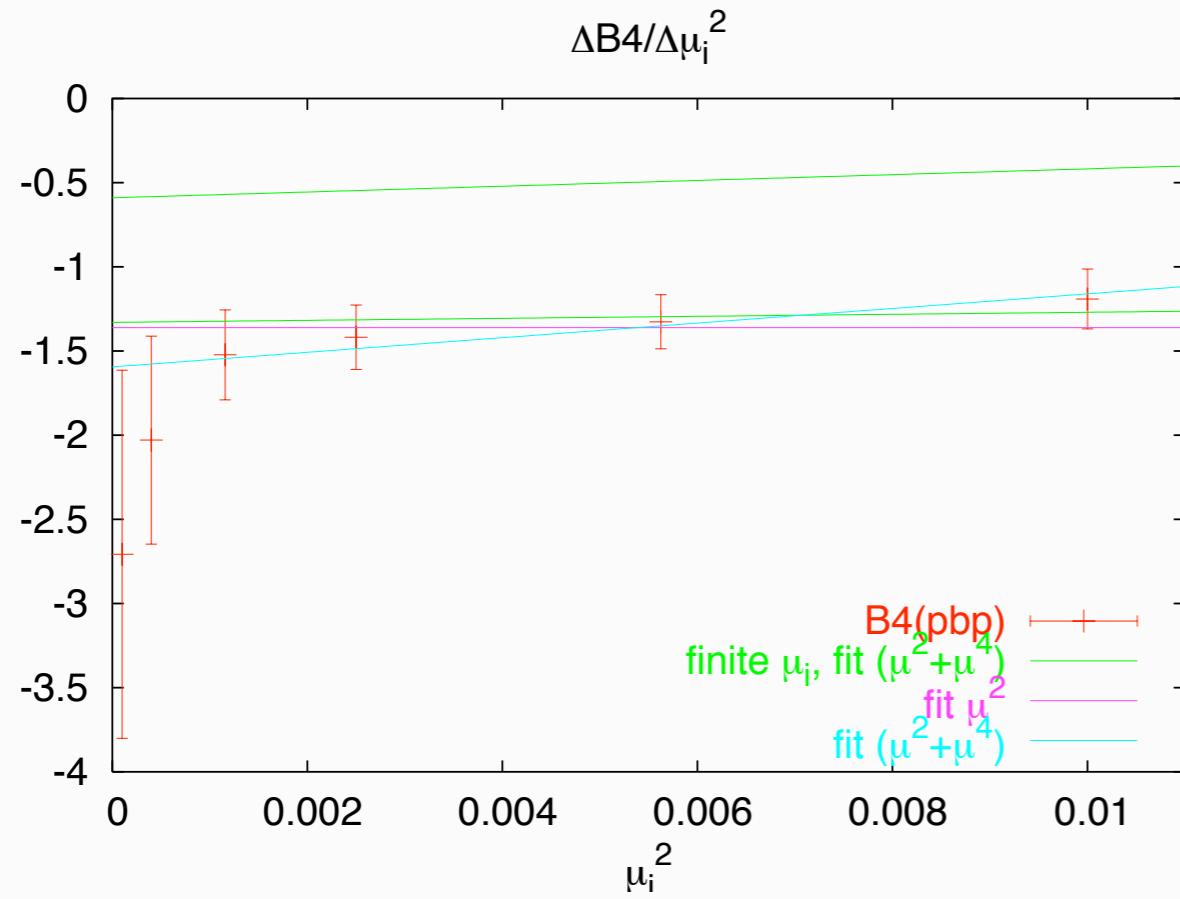
Numerical results for $N_f = 3, N_t = 4$

de Forcrand, O.P. 08

unimproved staggered fermions, RHMC algorithm

I. imag. μ : $8^3 \times 4$, 42 pairs $(am, a\mu_i)$ > 20 million traj., 18 unconstrained dof's in fits

II: deriv. at $\mu = 0$: $8^3, 12^3 \times 4$ $m_\pi L \gtrsim 3, 4.5$ > 5 million, 0.5 million traj.

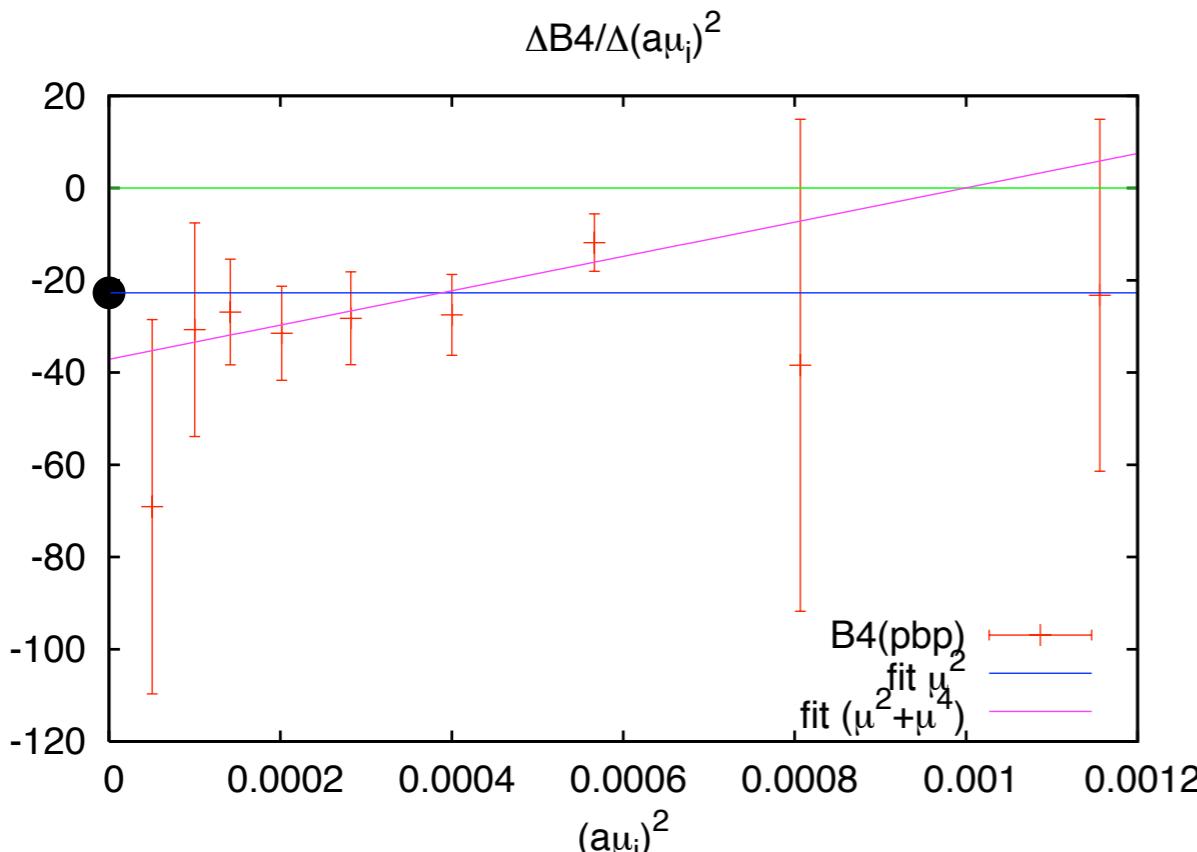


$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3) \left(\frac{\mu}{\pi T} \right)^2 - 47(20) \left(\frac{\mu}{\pi T} \right)^4 - \dots$$

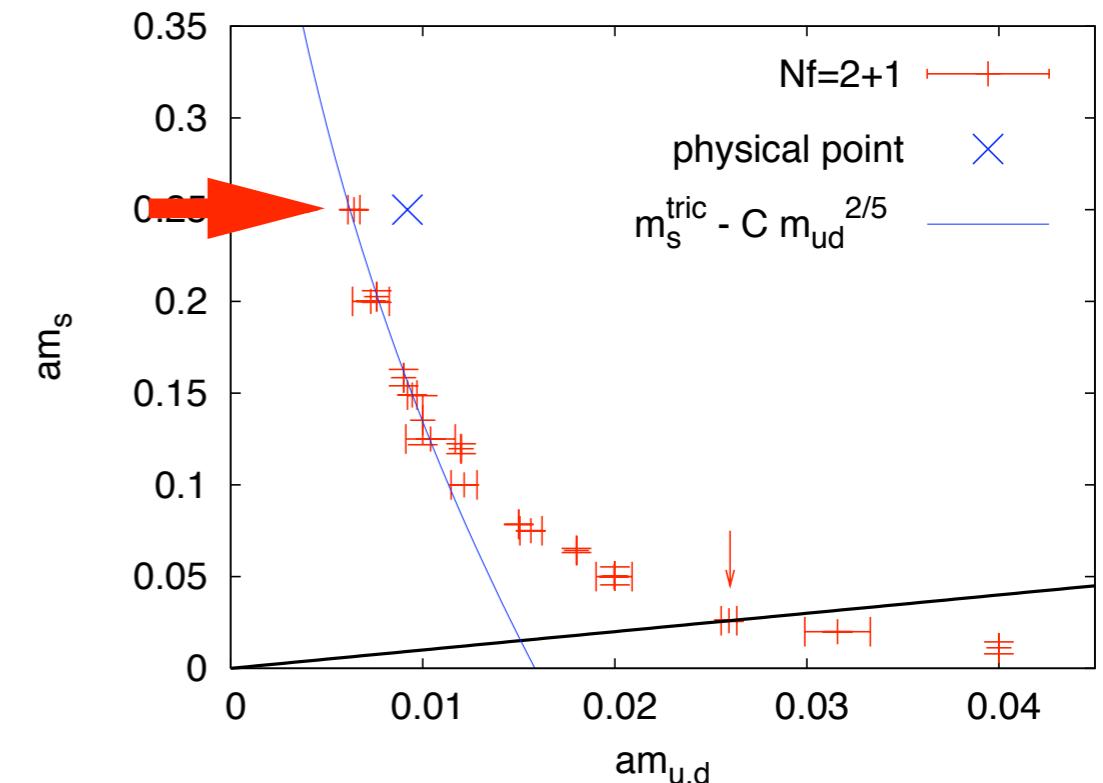
Exotic scenario!

Non-degenerate quark masses $N_f = 2 + 1, N_t = 4$

de Forcrand, O.P.09 with CERN-IT Grid computing



$$\frac{m_{u,d}^c(\mu)}{m_{u,d}^c(0)} = 1 - 24(11) \left(\frac{\mu}{\pi T} \right)^2 - \dots$$



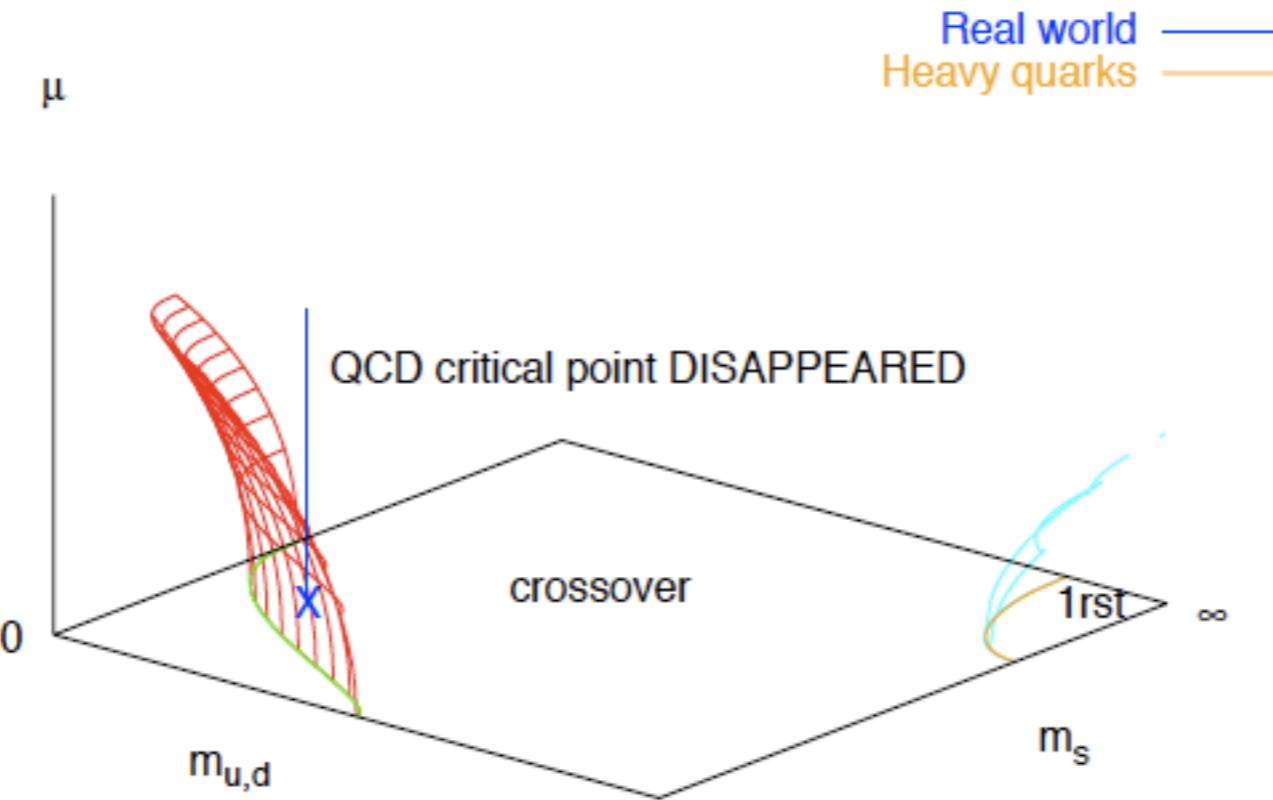
fix m_s to physical
 $m_{u,d}$ lighter than in nature

- $16^3 \times 4, am_s = 0.25, am_{u,d} = 0.005$ $(m_\pi L = 3.4)$

- $O(10^6)$ trajectories, 300 CPU-years by **Grid computing**
 $O(10^3)$ independent single-CPU threads of $O(10^3)$ trajectories each



On coarse lattice: exotic scenario



Weakening of p.t. with chemical potential also for:

-Heavy quarks

de Forcrand, Kim, Takaishi 05

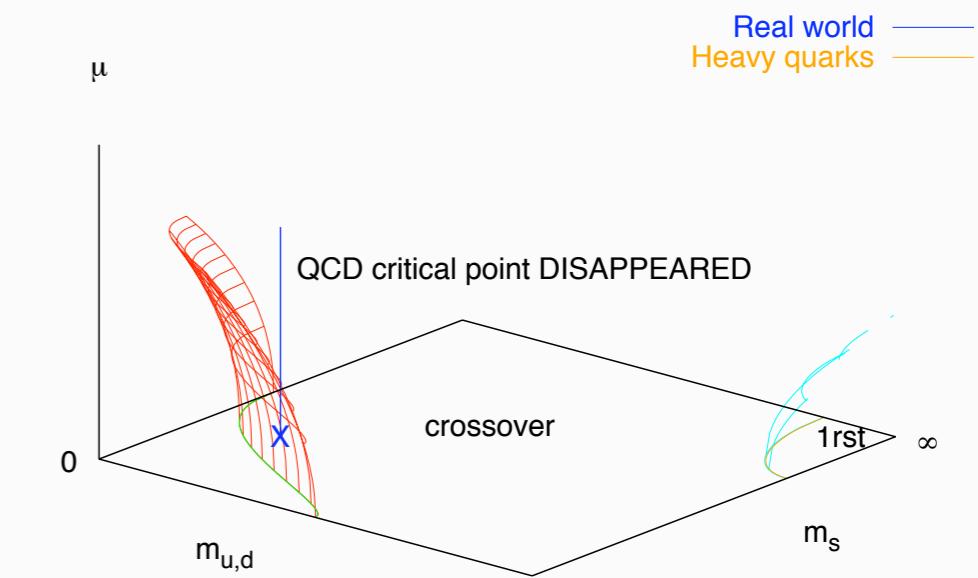
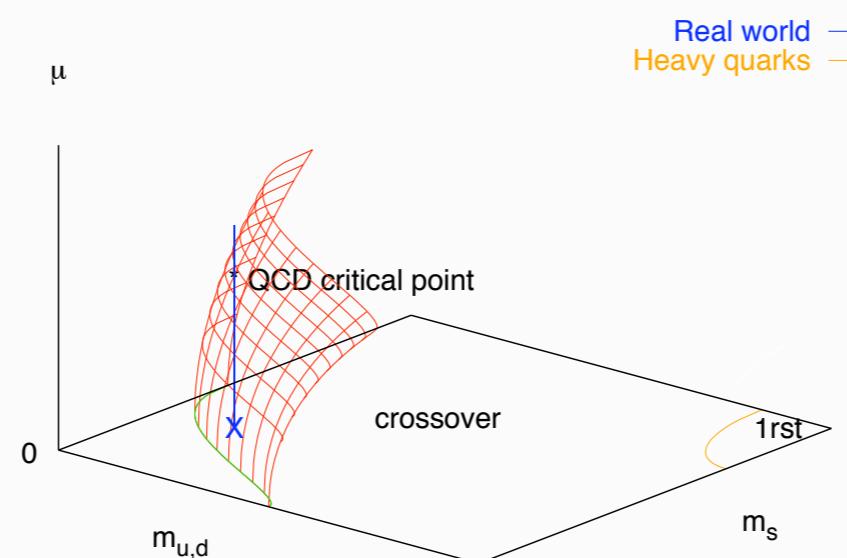
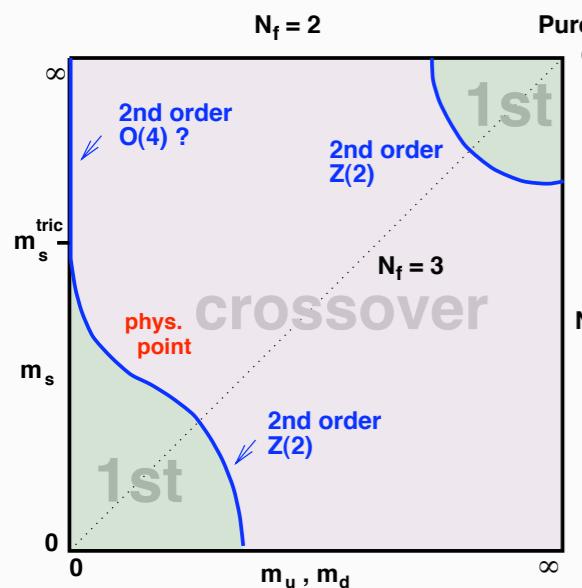
-Light quarks with finite isospin density

Kogut, Sinclair 07,
de Forcrand, Stephanov, Wenger

-Model studies PNJL, Sigma-model

Fukushima 08, Bowman, Kapusta 08

Finite density: chiral critical line → critical surface



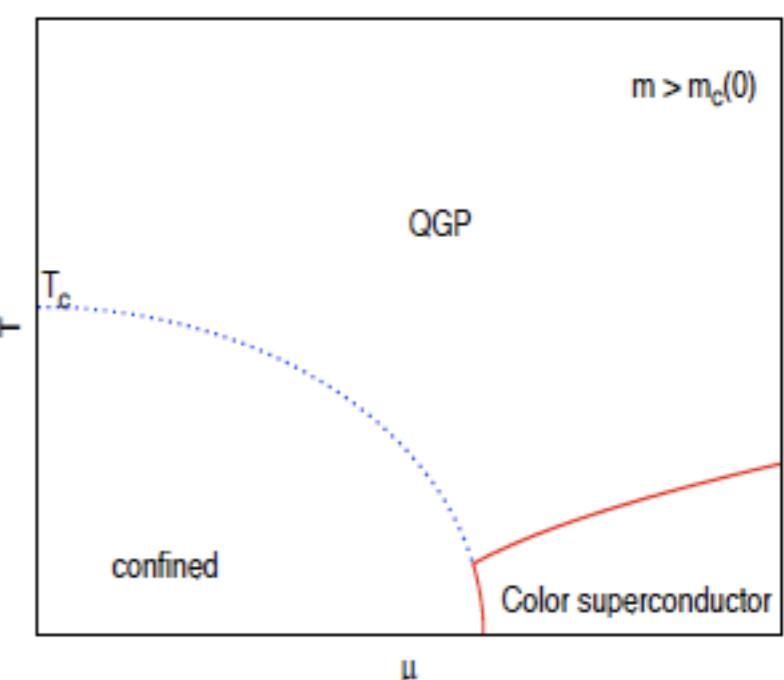
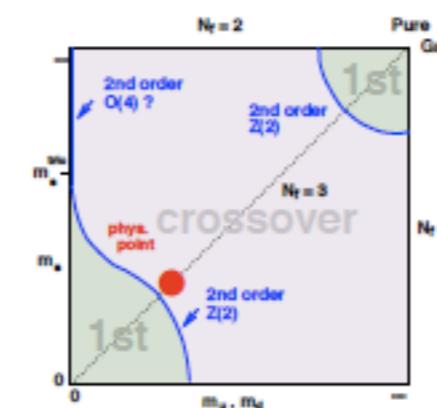
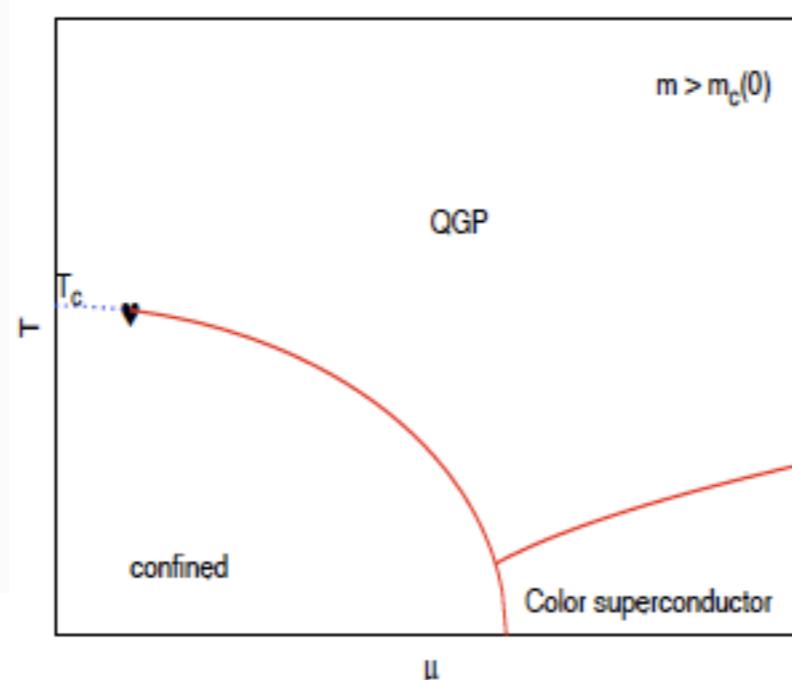
$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} \mathbf{c}_k \left(\frac{\mu}{\pi T} \right)^{2k}$$

$$c_1 > 0$$

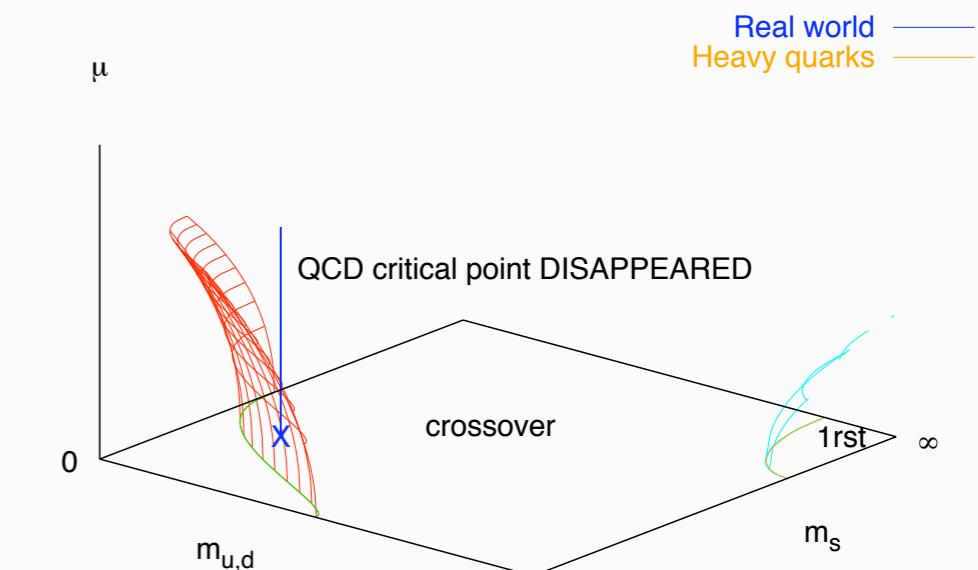
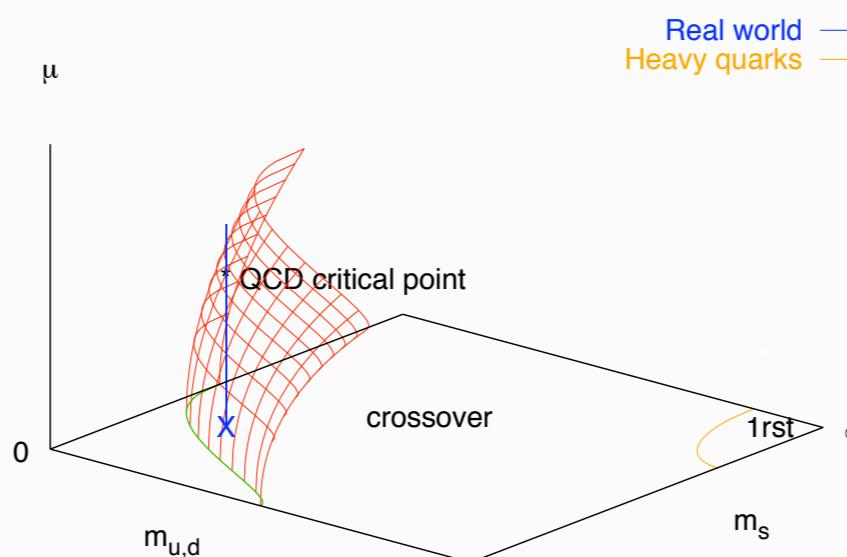
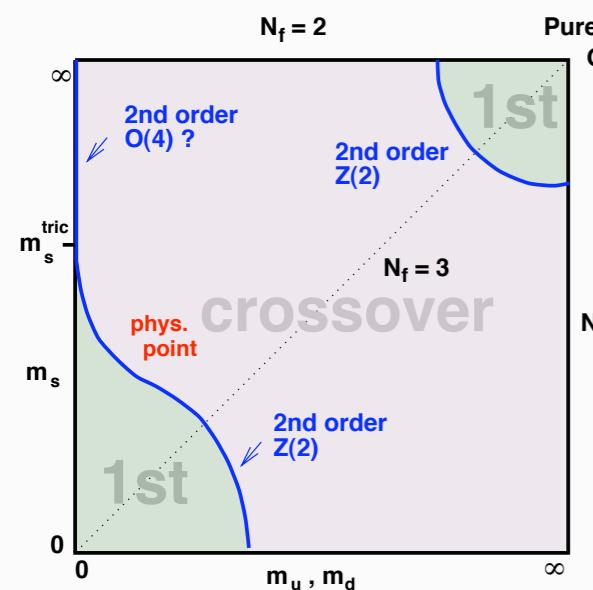
$$c_1 < 0$$

Standard scenario

Exotic scenario



Finite density: chiral critical line → critical surface



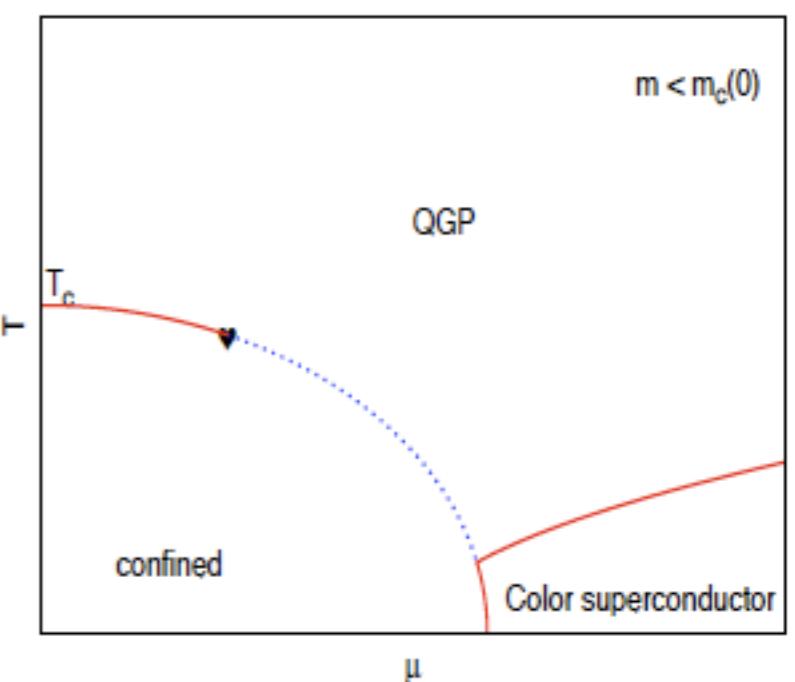
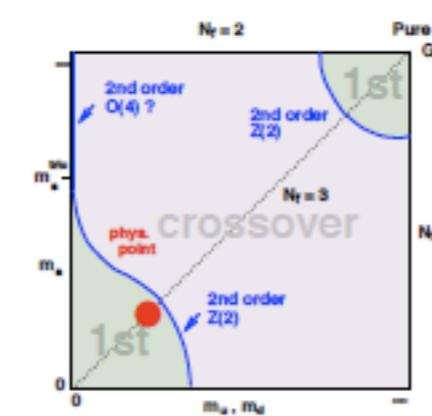
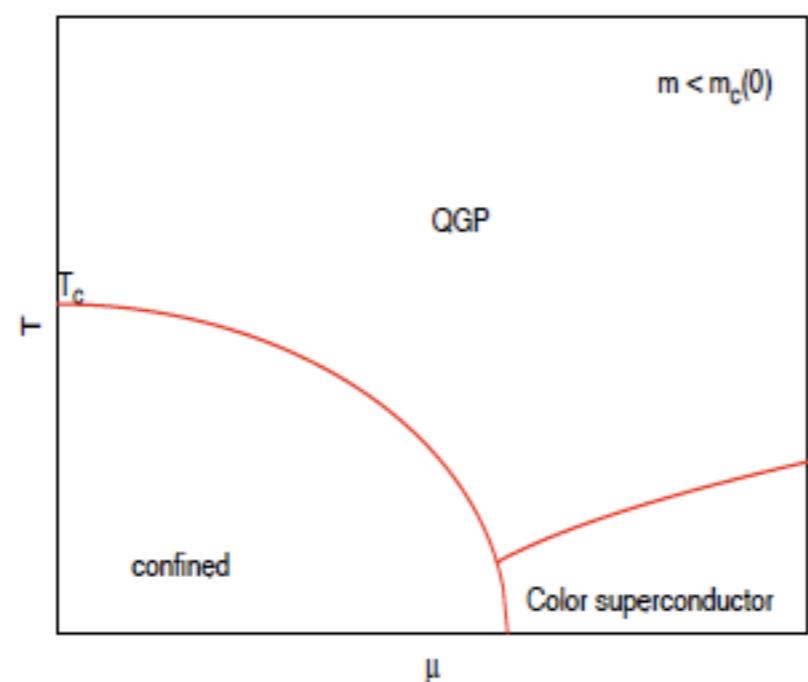
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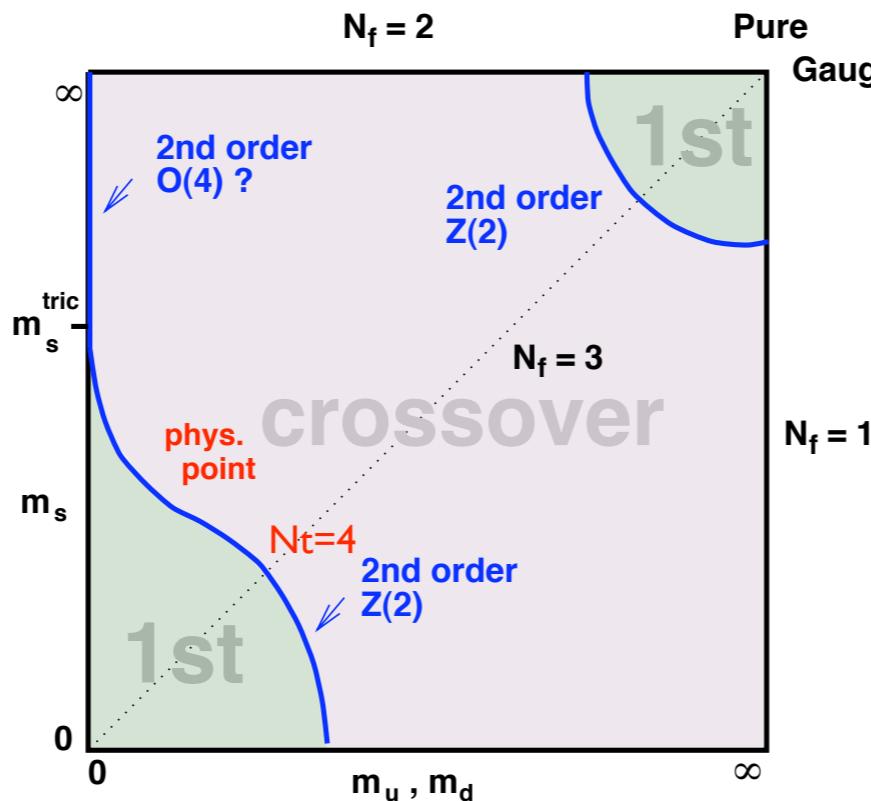
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Standard scenario

Exotic scenario



Towards the continuum: $N_t = 6, a \sim 0.2 \text{ fm}$

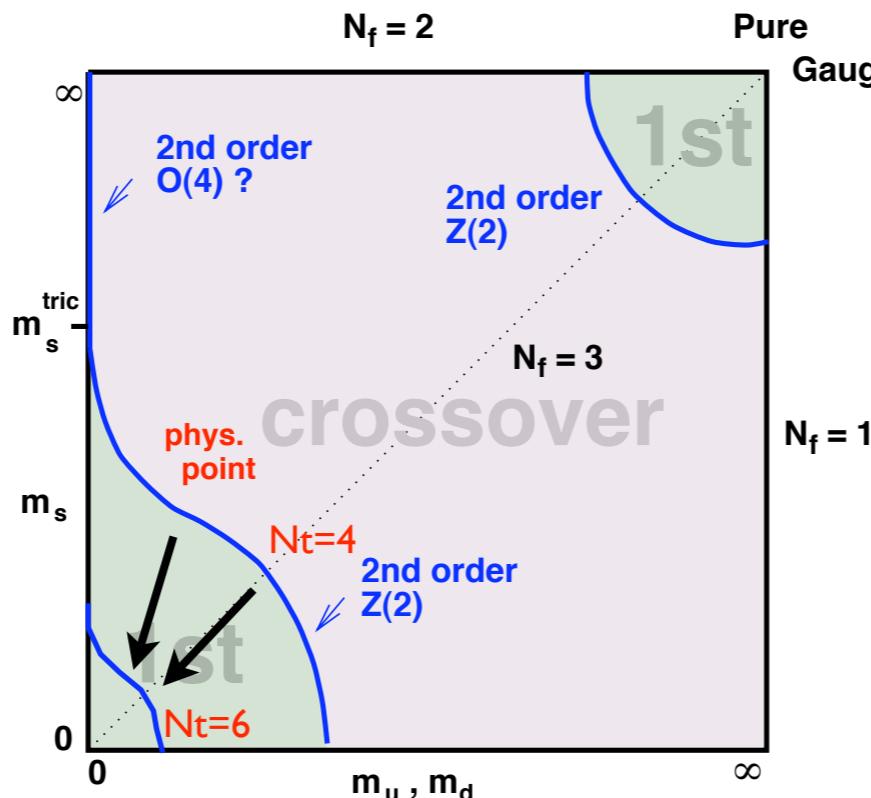


$$\frac{m_\pi^c(N_t = 4)}{m_\pi^c(N_t = 6)} \approx 1.77 \quad N_f = 3$$

de Forcrand, Kim, O.P. 07
Endrodi et al 07

- Physical point deeper in crossover region as $a \rightarrow 0$
- Cut-off effects stronger than finite density effects!
- tri-critical strange mass shrinking:
which critical surface will intersect with physical point?

Towards the continuum: $N_t = 6, a \sim 0.2 \text{ fm}$

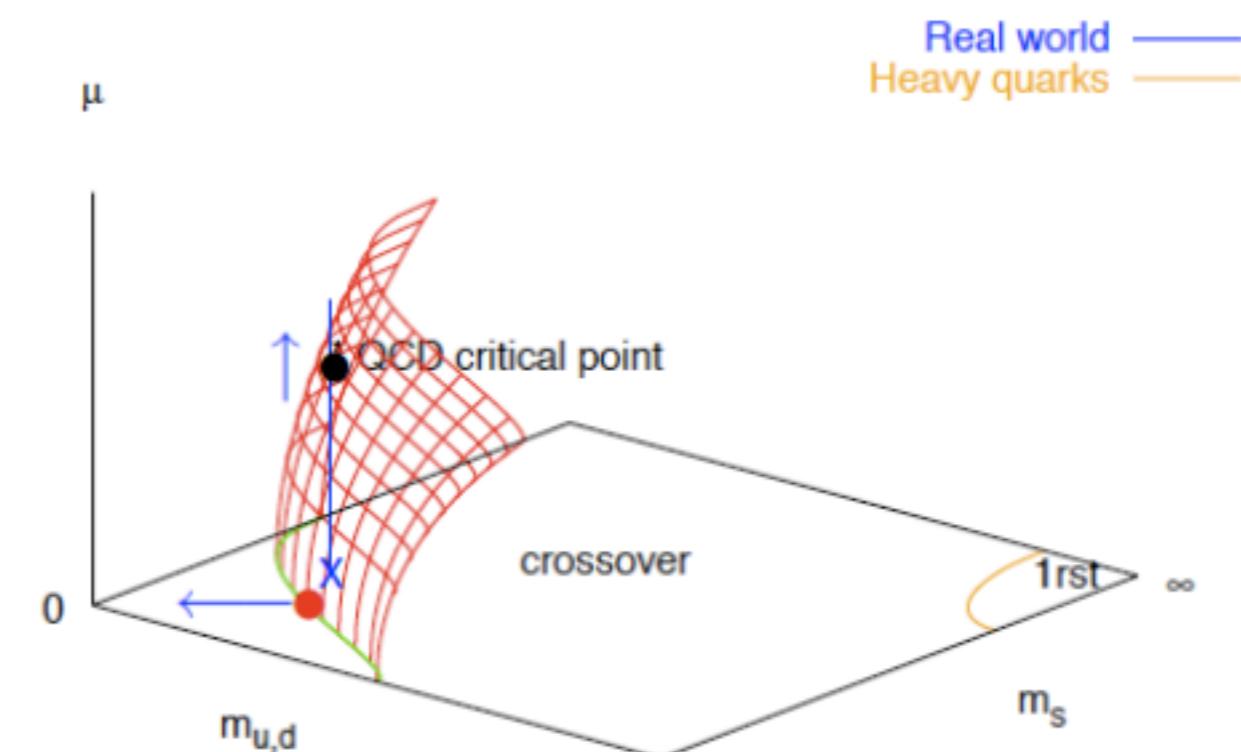
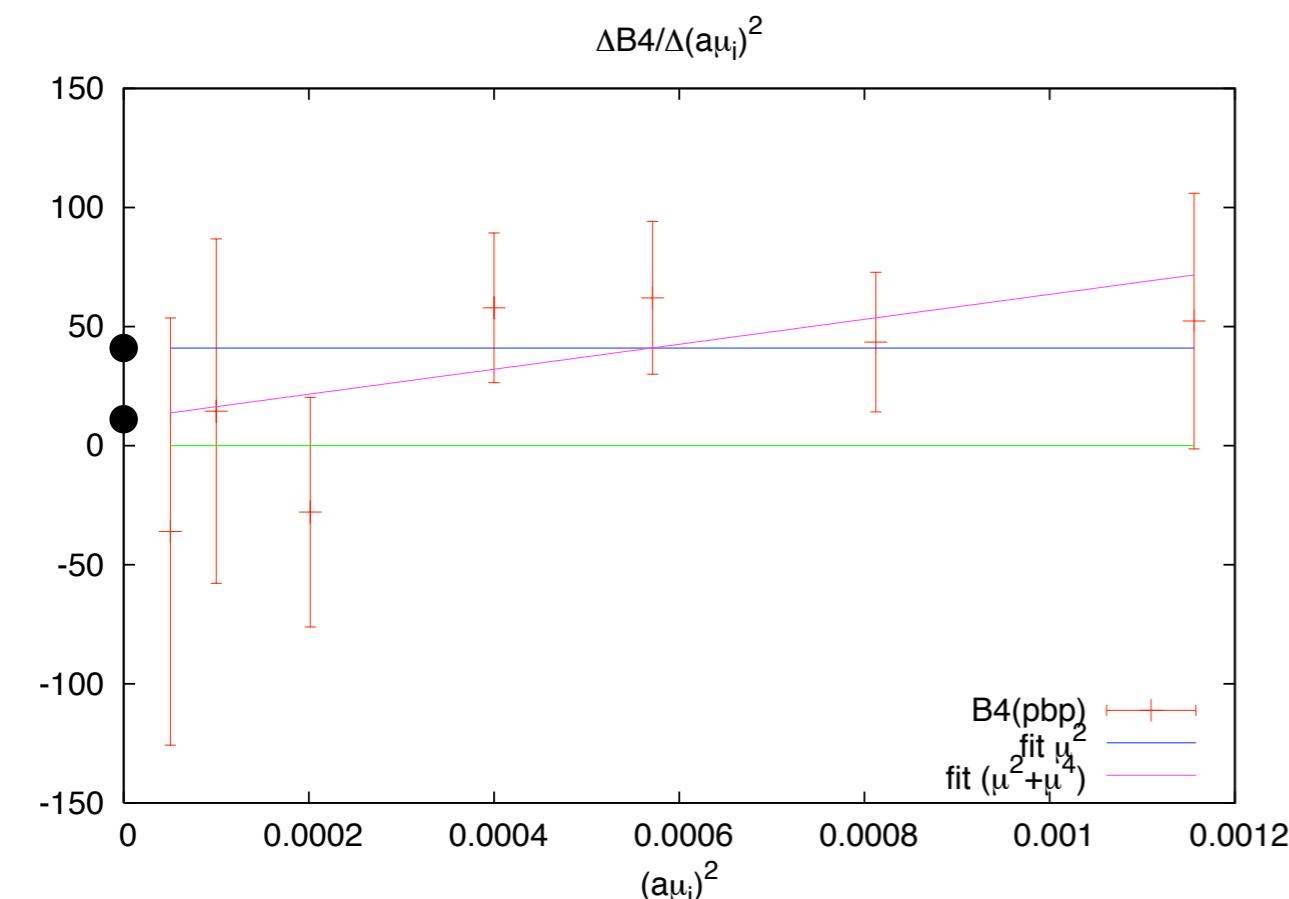


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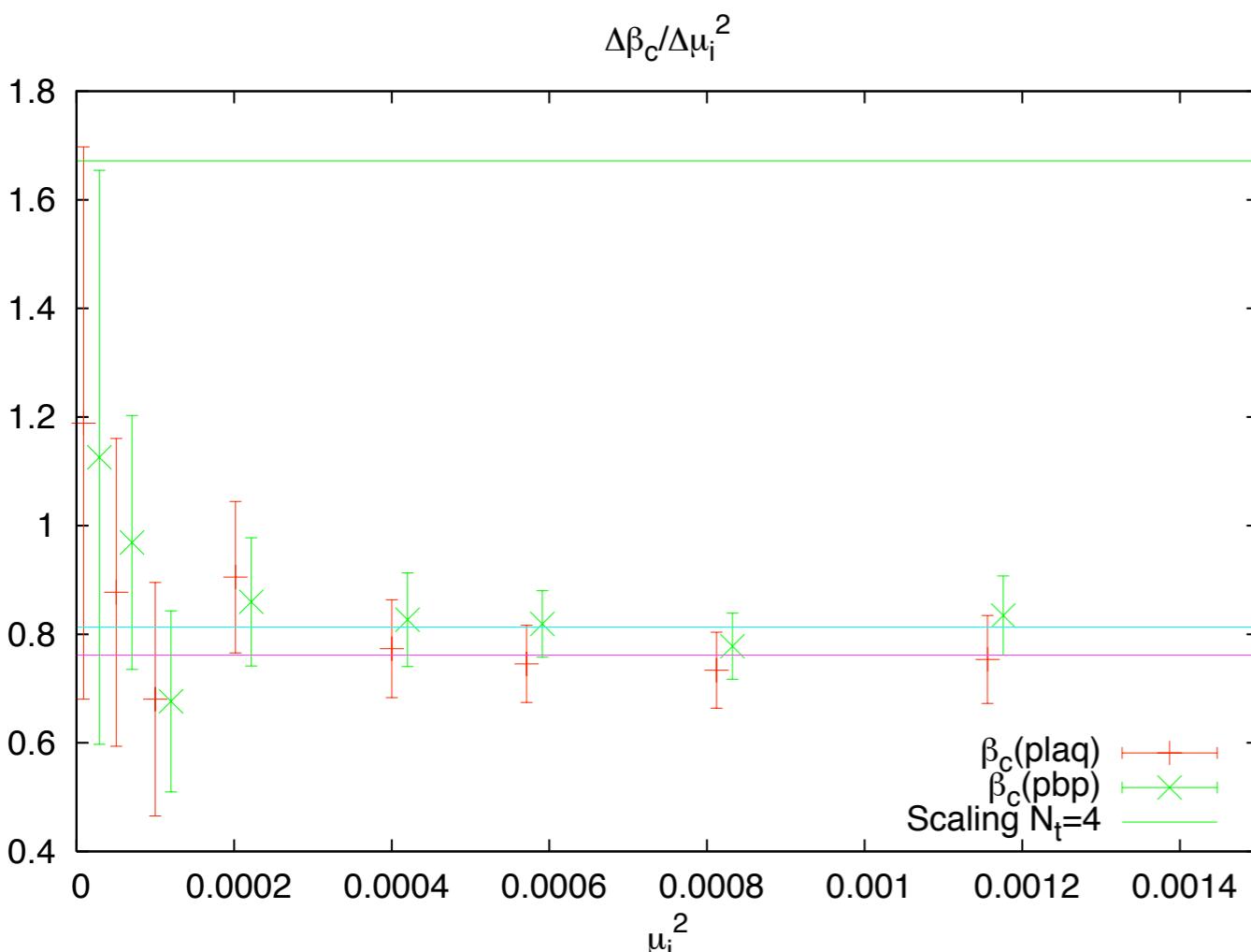
- Physical point deeper in crossover region as $a \rightarrow 0$
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...and the (preliminary) curvature on $N_t = 6, a \sim 0.2$ fm



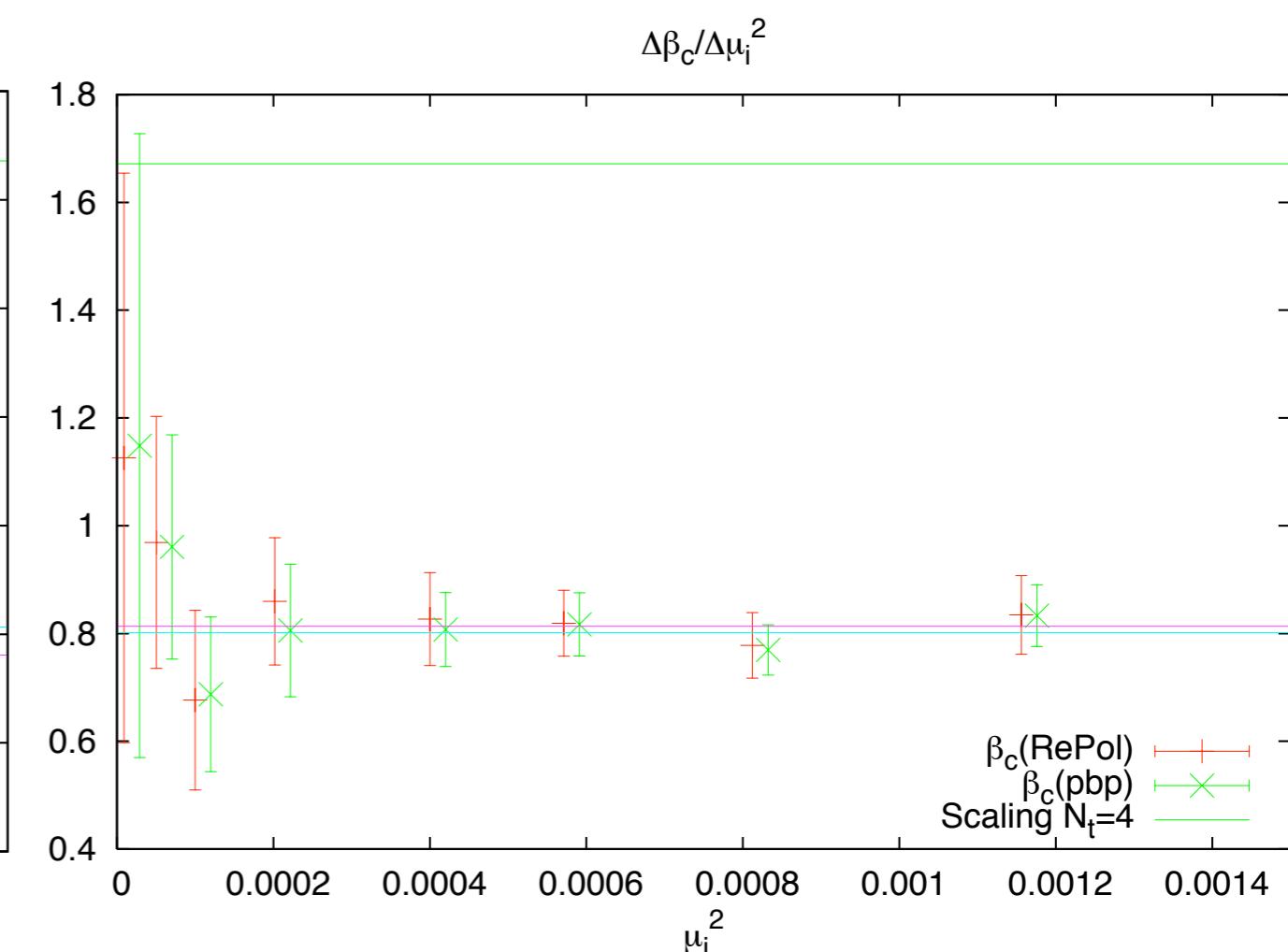
- LO, NLO extrapolation, not yet stable
- Curvature of crit. surface may change sign as function of lattice spacing
- So far too small to make up for shift of zero density baseline
small curvature=strong quark mass dependence of end point!

By-product: curvature of ‘critical temperature’



chiral condensate and plaquette

● No signal for NLO term



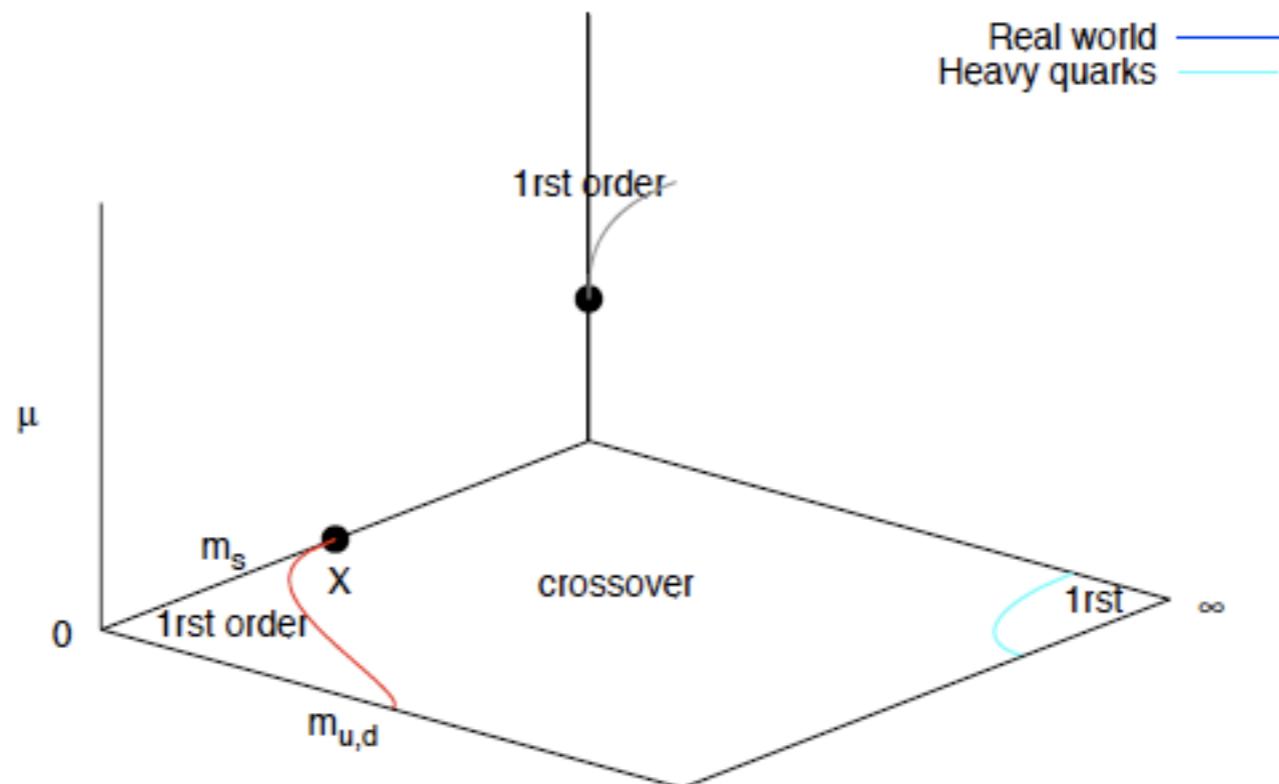
chiral condensate and Polyakov loop

● Curvature nearly the same for all observables

The interplay of Nf=2 and Nf=2+1

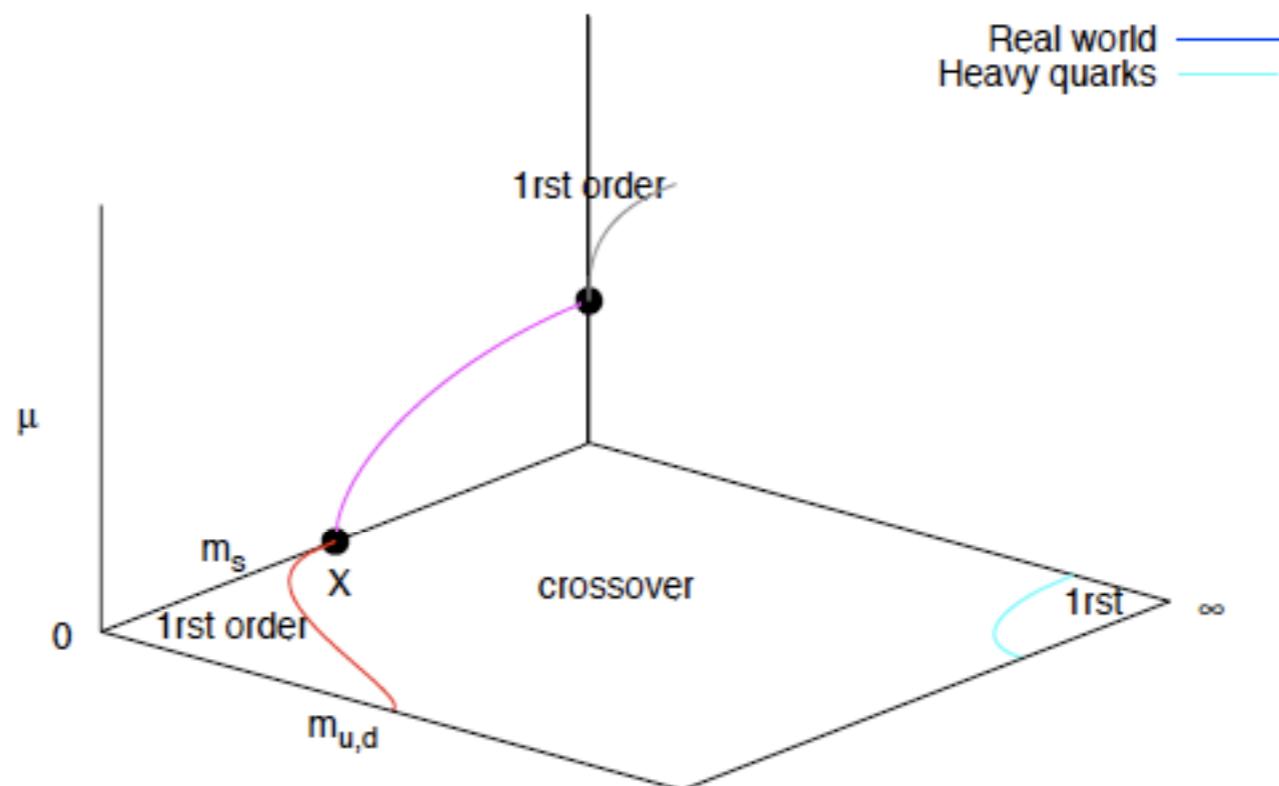
- $O(4)$ transition for 2 massless flavors
⇒ tricritical points ($m_{u,d} = 0, m_s = \infty, \mu = \mu^*$) and ($m_{u,d} = 0, m_s = m_s^*, \mu = 0$)

Pisarski & Wilczek



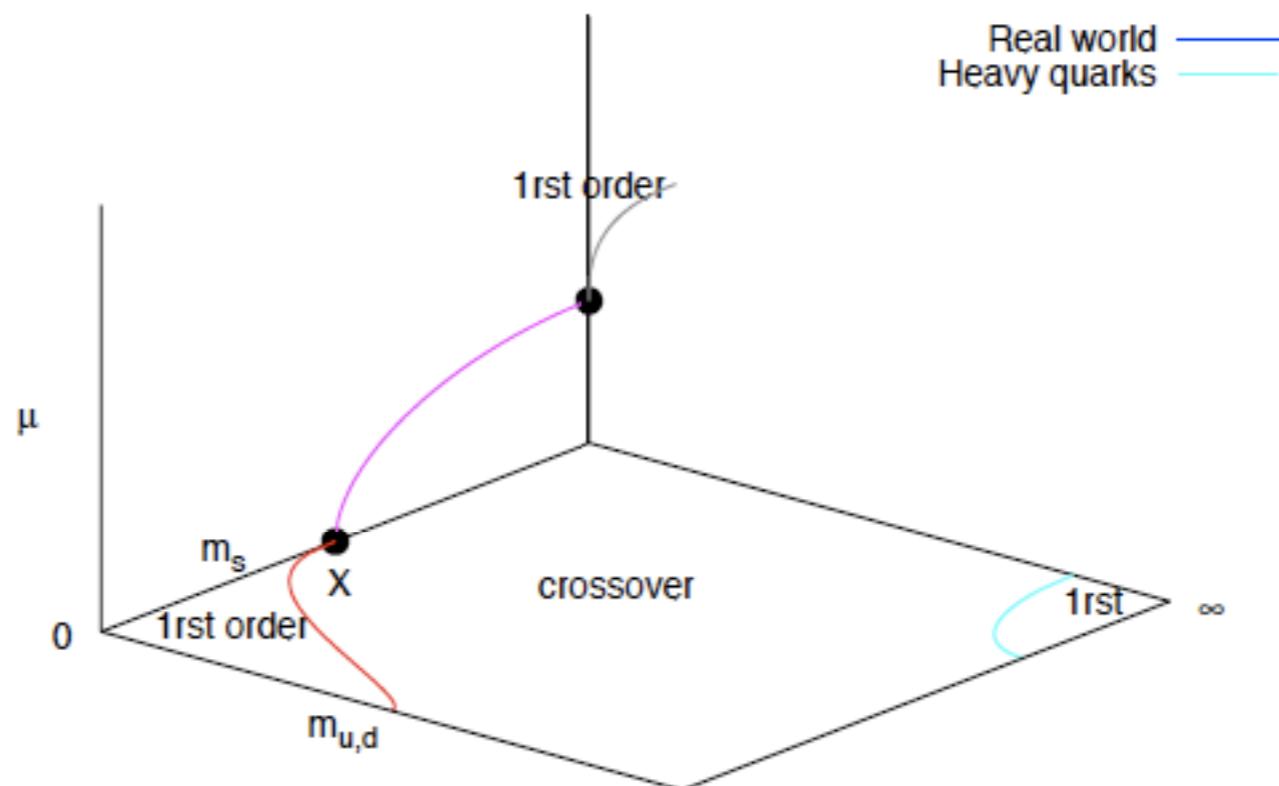
The interplay of $N_f=2$ and $N_f=2+1$

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- ⇒ tricritical points ($m_{u,d} = 0, m_s = \infty, \mu = \mu^*$) and ($m_{u,d} = 0, m_s = m_s^*, \mu = 0$)
- $N_f = 2$ and $N_f = 2 + 1$ analytically connected



The interplay of $N_f=2$ and $N_f=2+1$

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- \Rightarrow tricritical points ($m_{u,d} = 0, m_s = \infty, \mu = \mu^*$) and ($m_{u,d} = 0, m_s = m_s^*, \mu = 0$)
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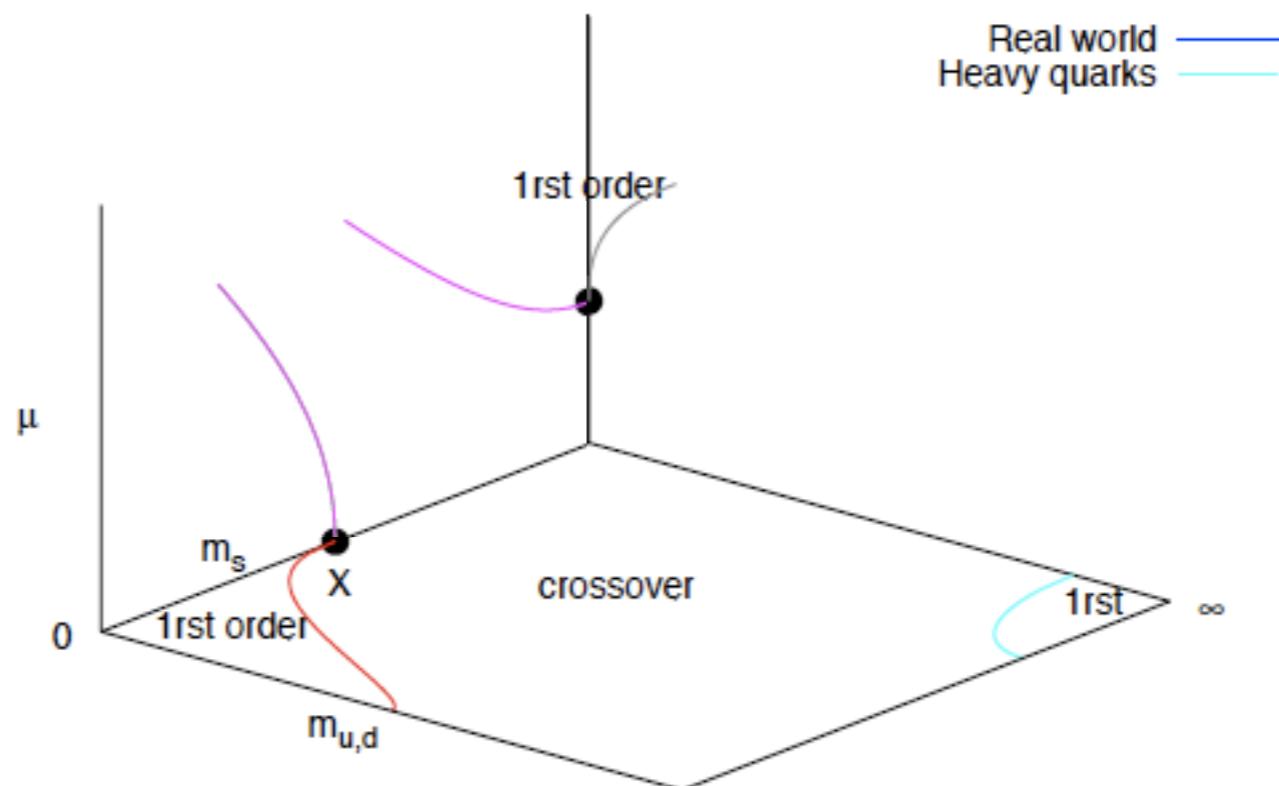
Critique:

- $O(4)$ if strong enough $U_A(1)$ anomaly, otherwise first-order

Chandrasekharan & Mehta

The interplay of $N_f=2$ and $N_f=2+1$

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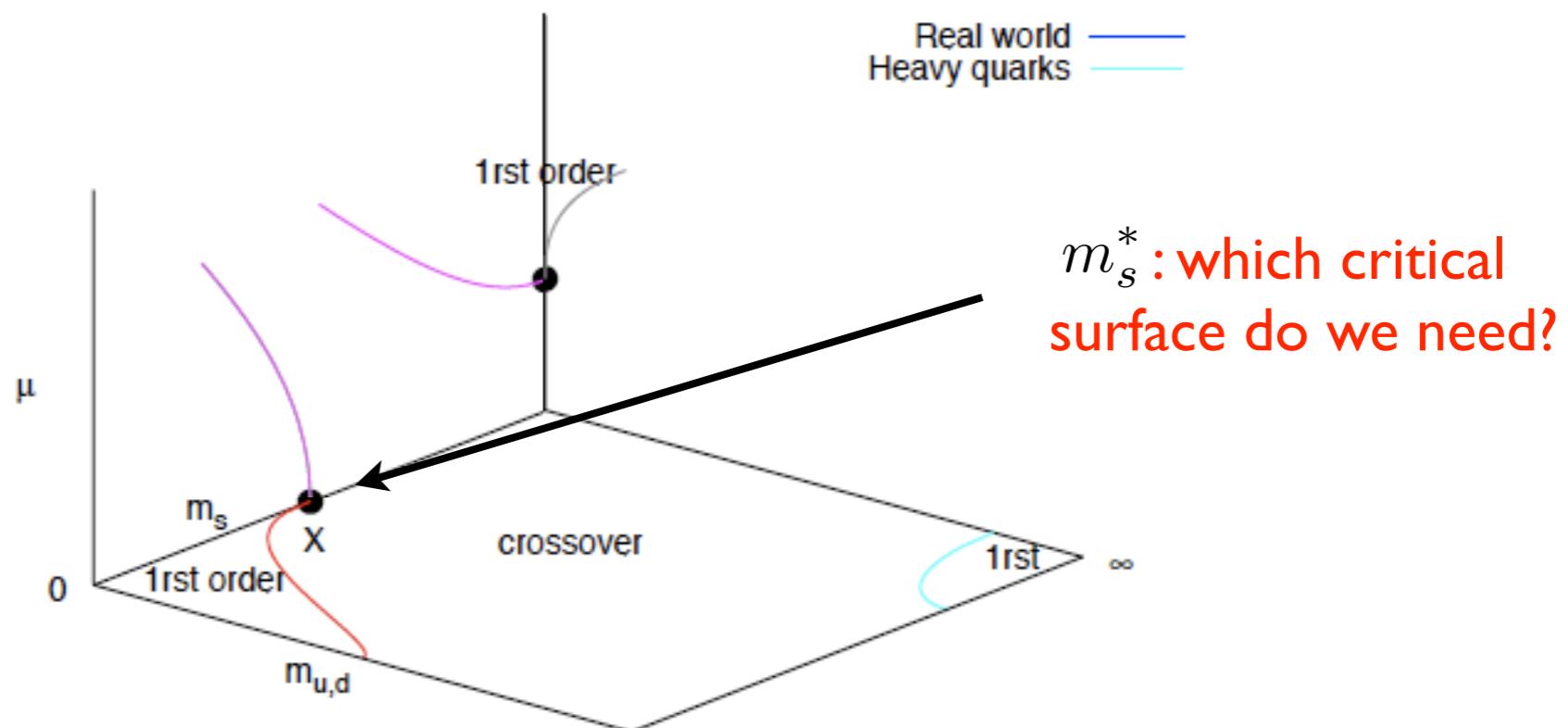
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The interplay of $N_f=2$ and $N_f=2+1$

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Pisarski & Wilczek

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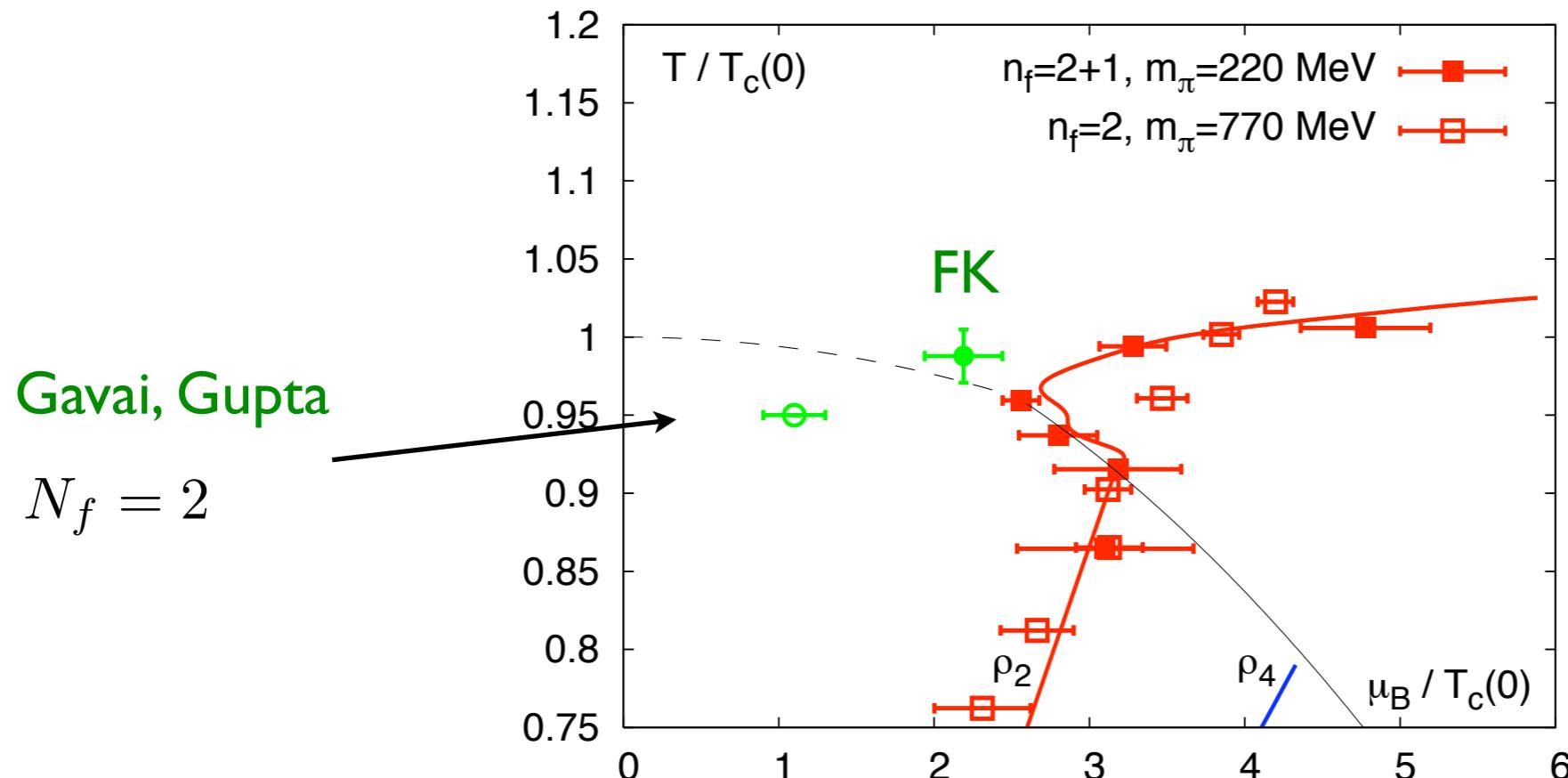
- $N_f = 2$ and $N_f = 2 + 1$ need not be connected

CEP from Taylor expansion

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Nearest singularity=radius of convergence $\frac{\mu_E}{T_E} = \lim_{n \rightarrow \infty} \sqrt{\left| \frac{c_{2n}}{c_{2n+2}} \right|}, \quad \lim_{n \rightarrow \infty} \left| \frac{c_0}{c_{2n}} \right|^{\frac{1}{2n}}$

Different definitions, agree only for $n \rightarrow \infty$ not $n=1,2,3$



Bielefeld-RBC
improved staggered
 $N_t = 4$

Check predictivity in a toy model

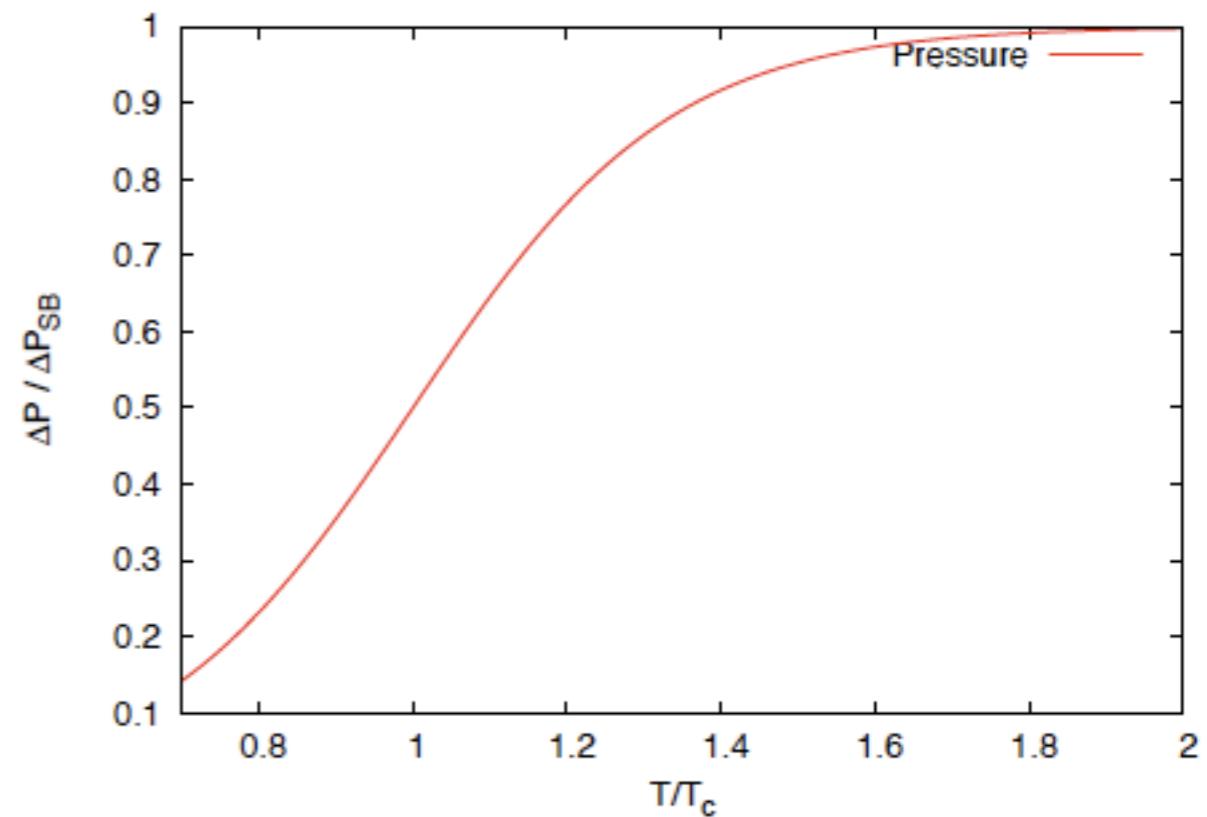
de Forcrand, Herrigel in progress

Ansatz:

$$\frac{\Delta p(\mu)}{T^4} = \frac{\Delta p_{SB}(\mu)}{T^4} + \ln \left(\cosh \left[\lambda \left(\frac{T}{T_c} - 1 \right) + \frac{\Delta p_{SB}(\mu)}{2T^4} \right] \right) + f \left(\frac{T}{T_c} \right)$$

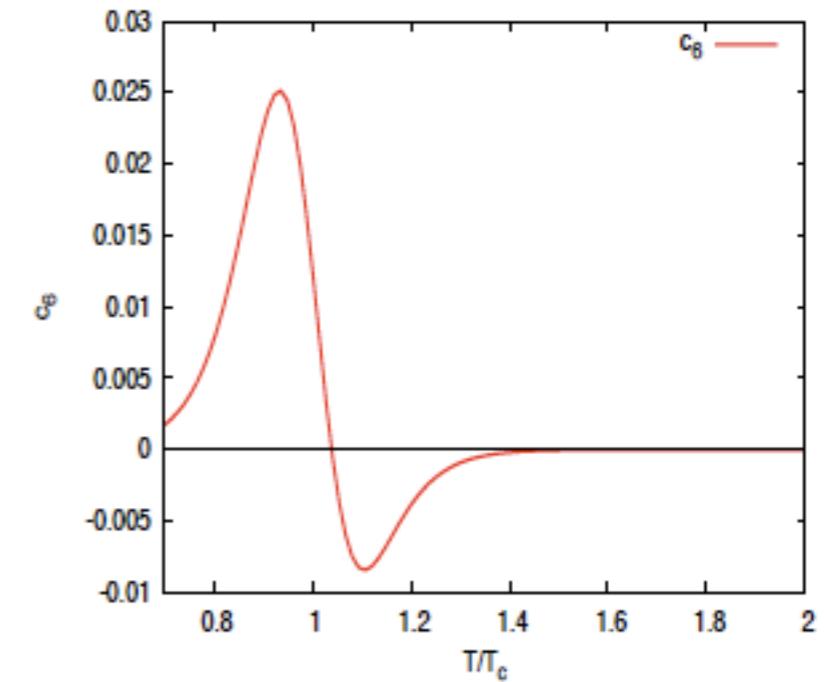
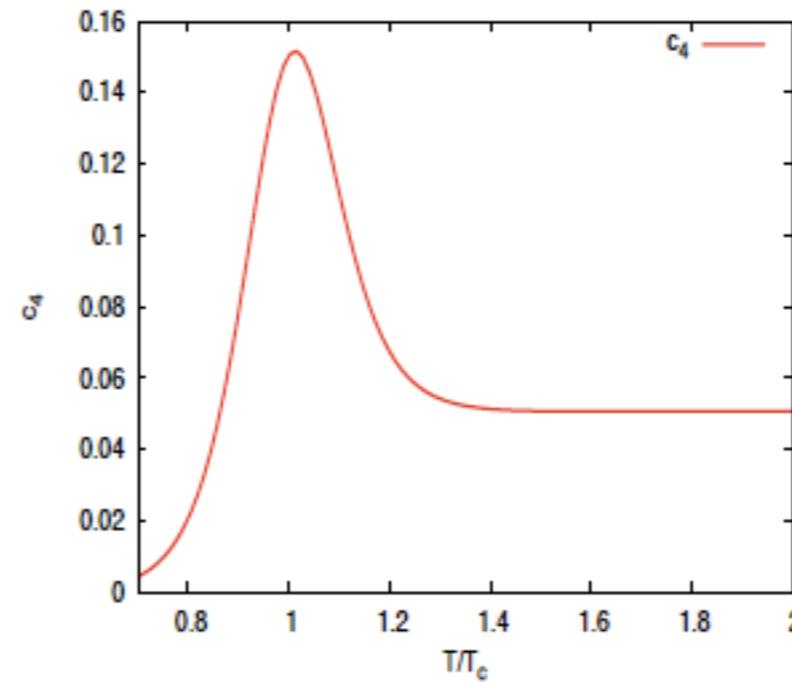
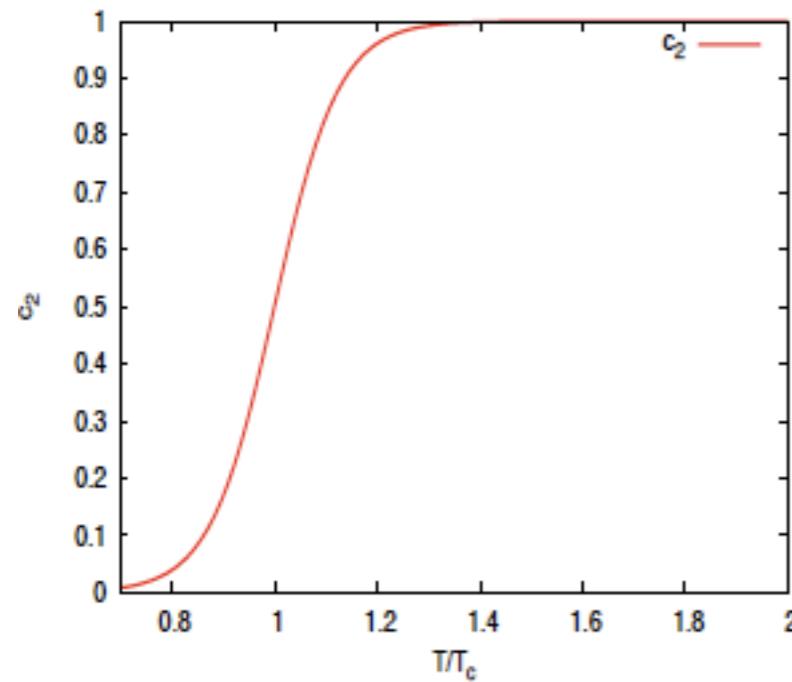
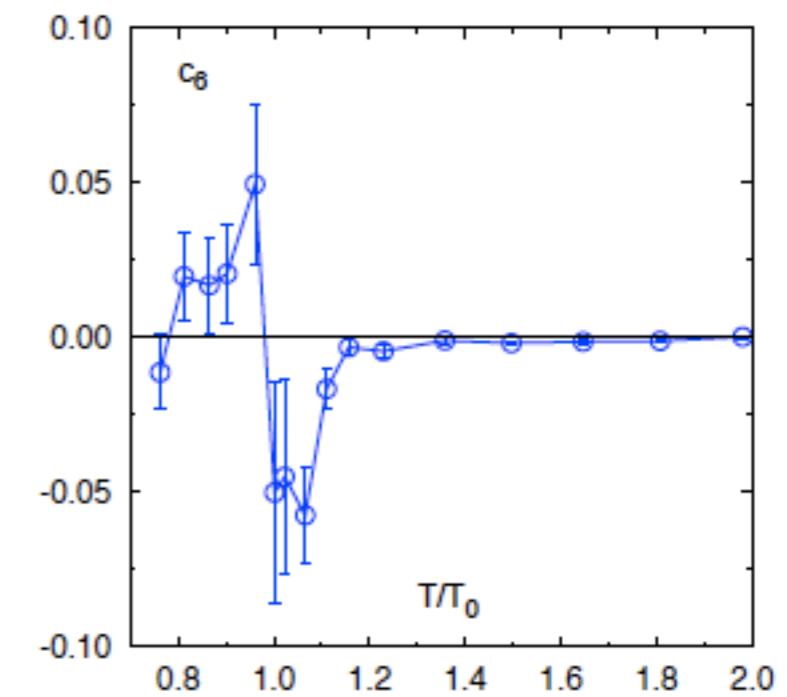
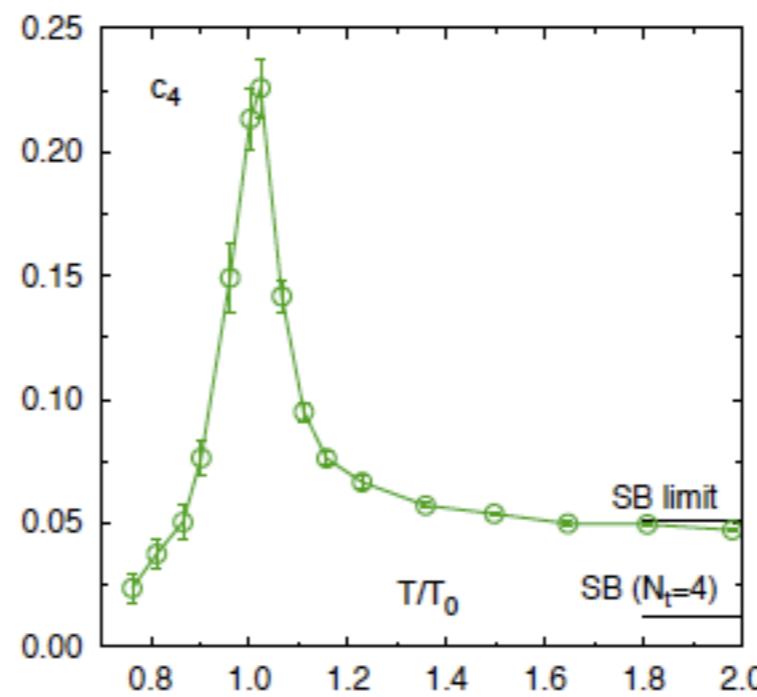
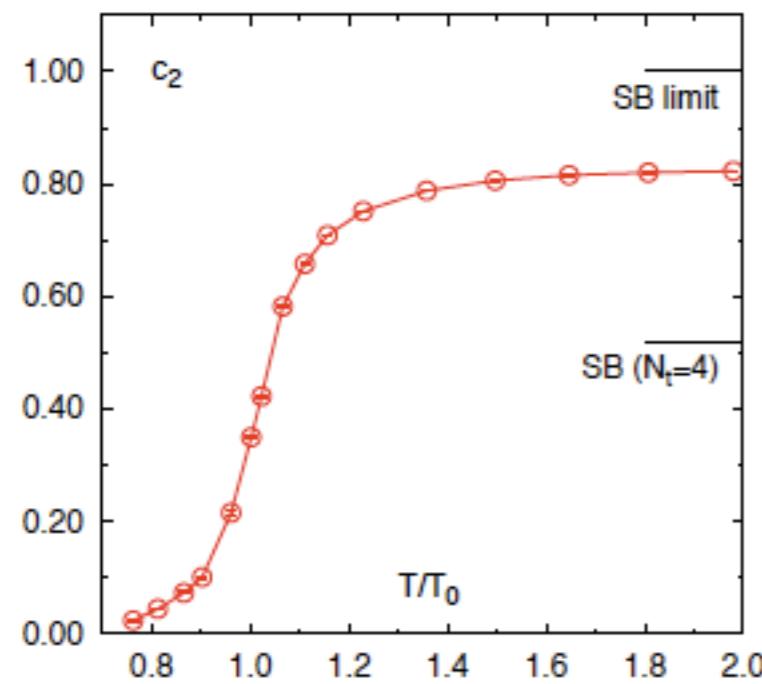
$$\Delta p_{SB}(\mu) = \frac{N_f}{2} \left(\left(\frac{\mu}{T} \right)^2 + \frac{1}{2\pi^2} \left(\frac{\mu}{T} \right)^4 \right)$$

- correct high, low T limits
- completely analytic, no p.t., no CEP
- λ controls width of crossover



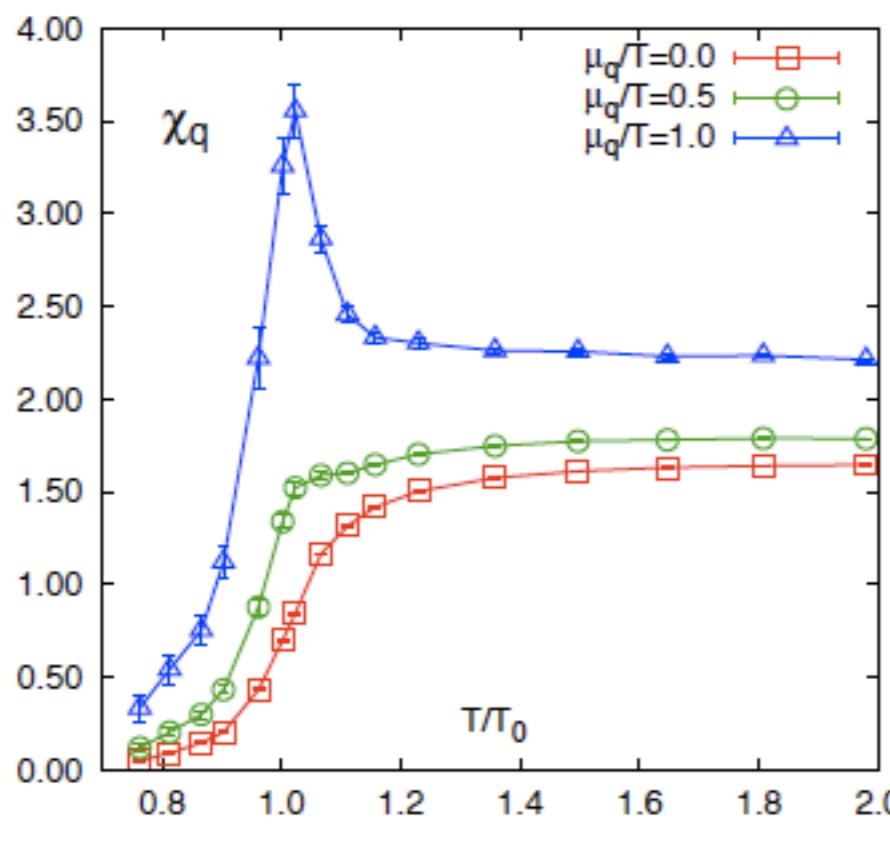
The coefficients:

Bielefeld-RBC

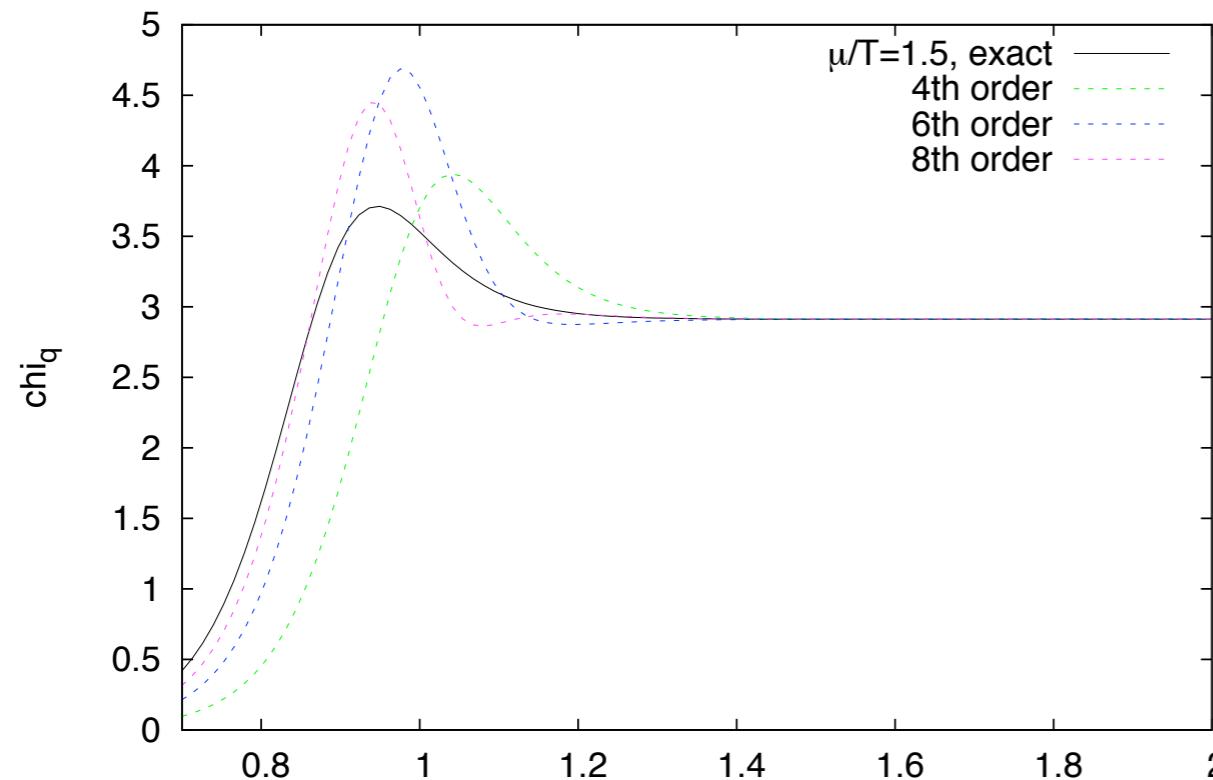
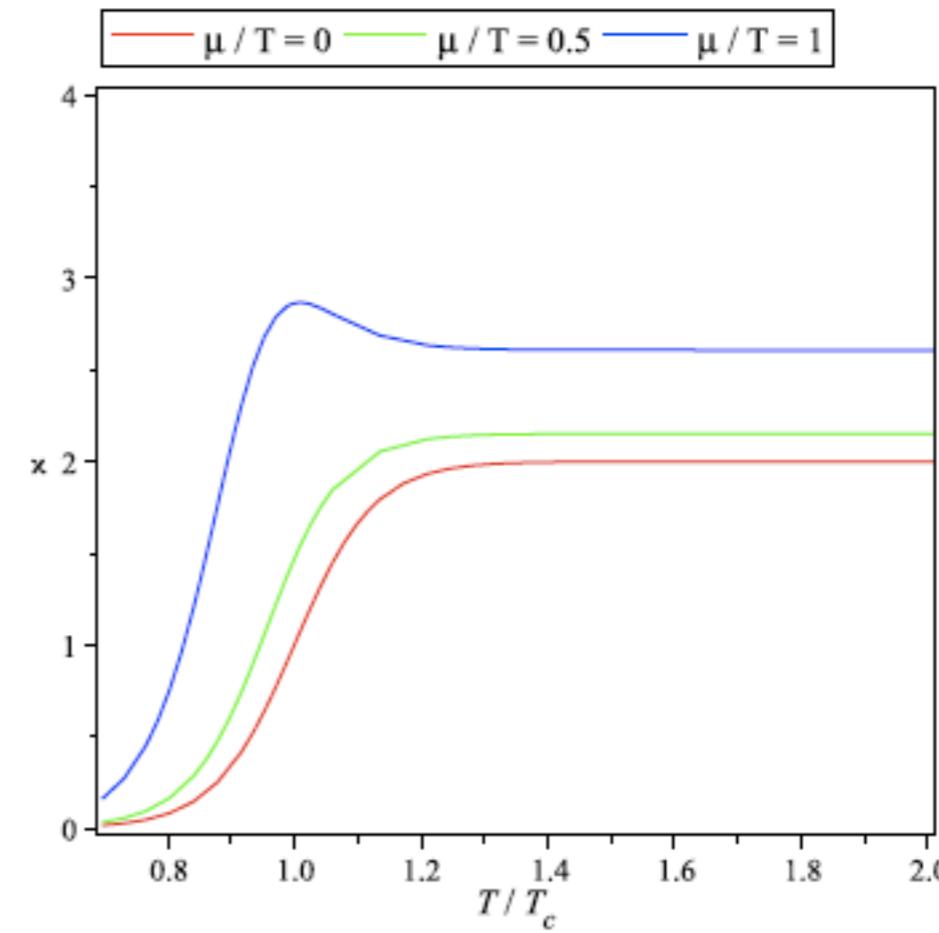


de Forcrand-Herrigel toy

Bielefeld-RBC



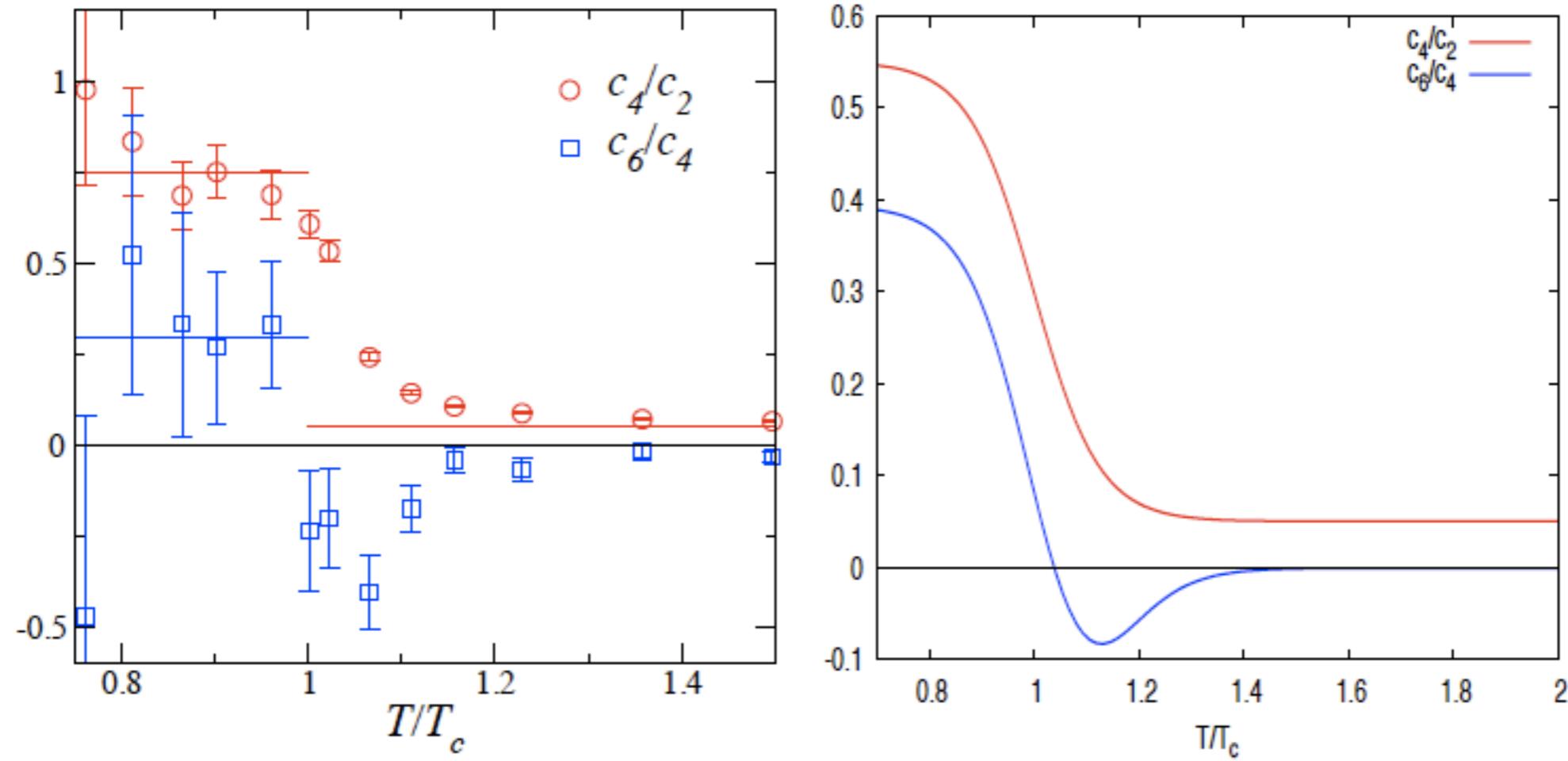
de Forcrand-Herrigel toy



Quark susceptibility peaks,
even without critical point!

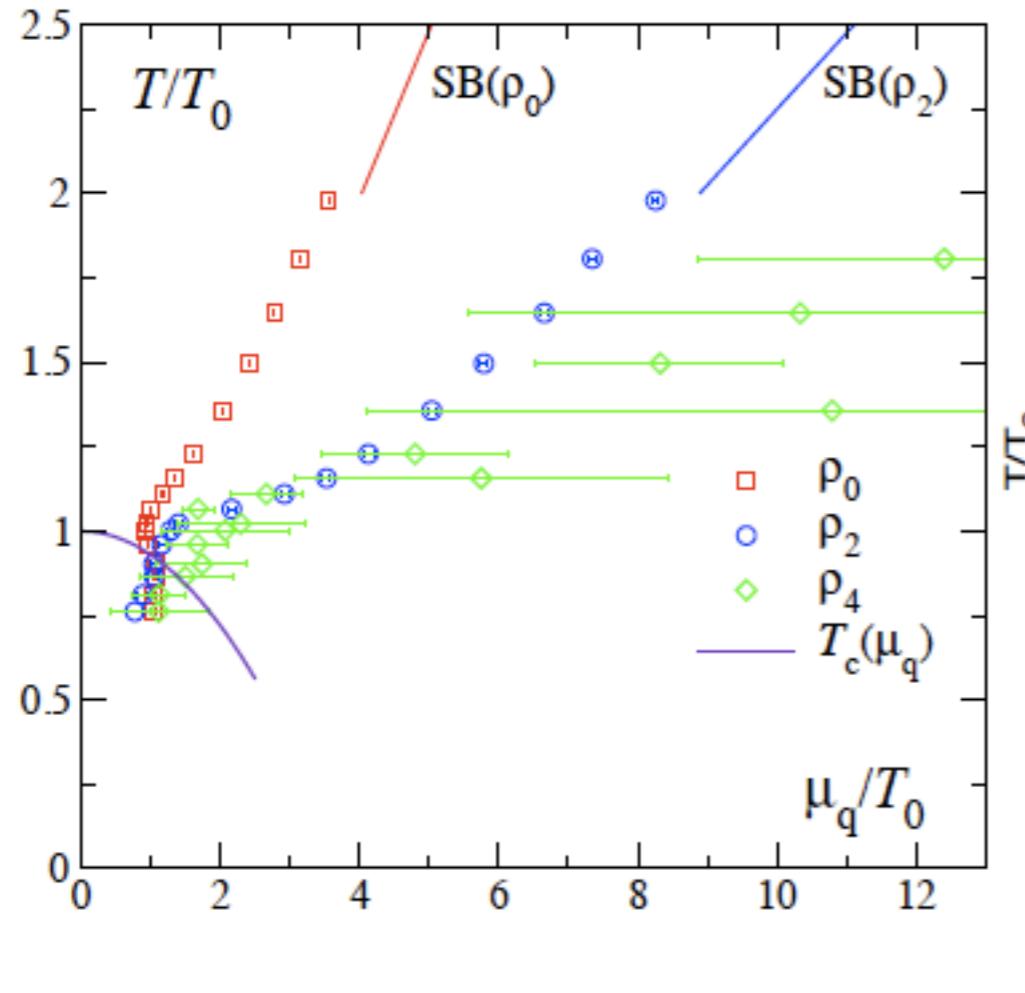
Bielefeld-RBC

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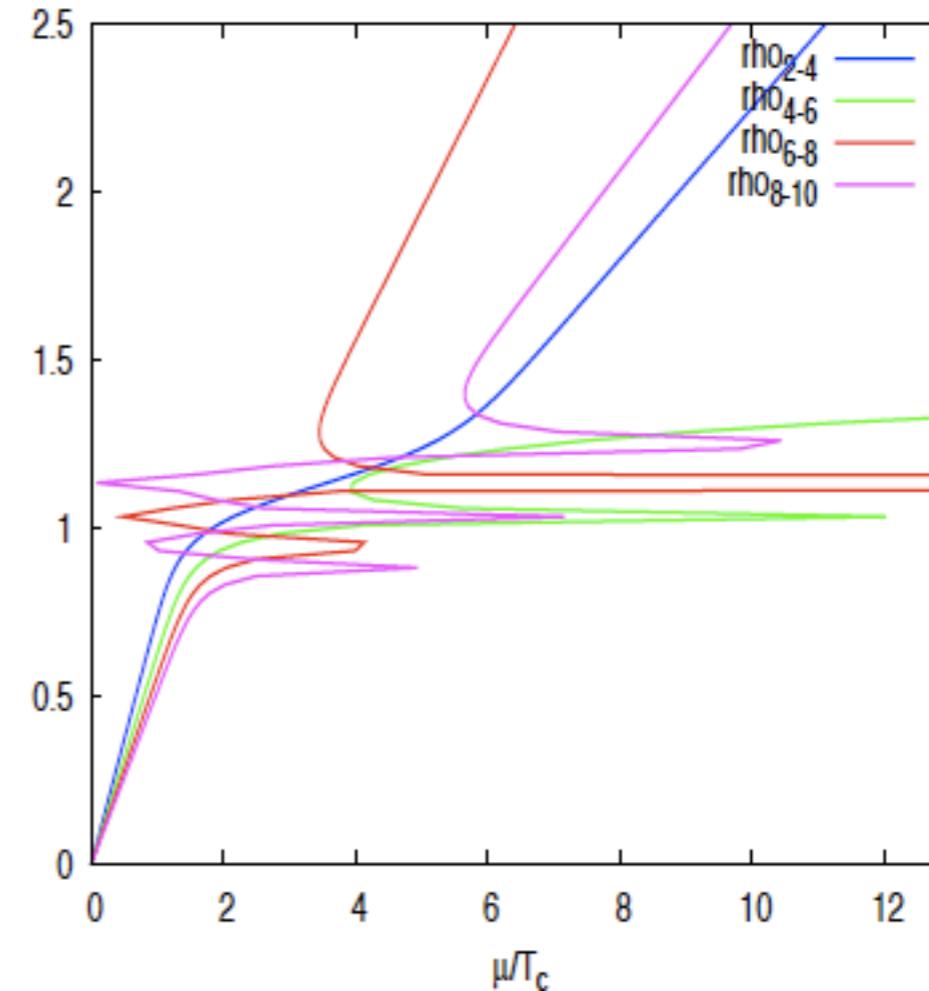


- Qualitative agreement below T_c although HRG not built-in

Bielefeld-RBC



de Forcrand-Herrigel toy



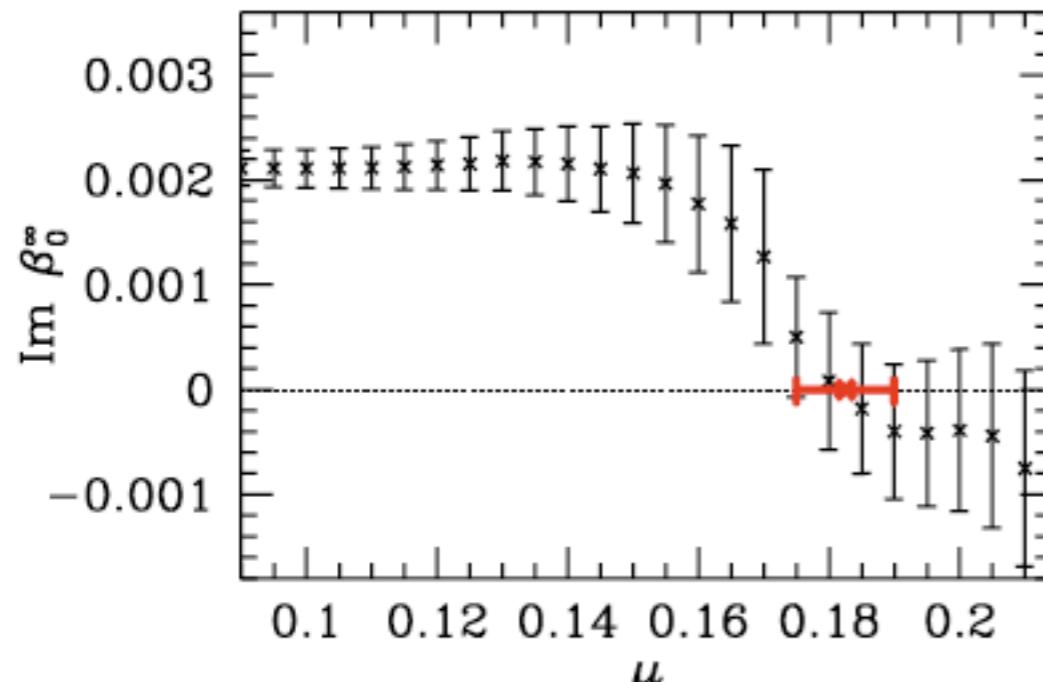
Can a few coefficients predict the existence of a CEP ????

Conclusions

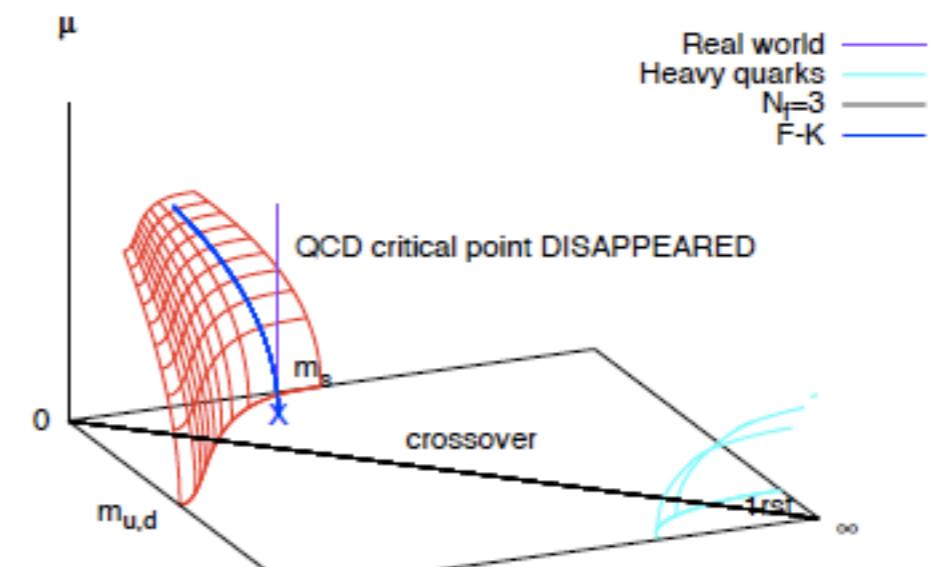
- CEP maximally difficult: very quark-mass and cut-off sensitive
- For lattices with $a \sim 0.3$ fm no chiral critical point for $\mu/T \lesssim 1$
- CEP scenario not yet clear: exploring uncharted territory!
- need to tackle with different approaches and for different Nf's

Comparison: CEP from reweighting $(\mu_E^q, T_E) = (120(13), 162(2)) \text{ MeV}$

$N_t = 4, N_f = 2 + 1$ physical quark masses, unimproved staggered fermions Fodor, Katz 04



Lee-Yang zero



- Abrupt change: physics or problem of reweighting? Splittorff 05, Han, Stephanov 08
- If it is physics, then many higher order Taylor coefficients might be required
- m/T kept fixed, quark masses **lighter** than physical at finite μ , favours 1st order
- Algorithmic step-size effects: favours 1st order, consistent with de Forcrand, O.P. 03